Don Lancaster's
Tech Musings

November, 1996

There sure was a lot of email over our simplified *trisamp* digital filter story from last month. These digital filters continue to amaze me.

Properly used in reasonable places, digitals can completely blow away all older analog designs. It is trivially easy to make a "gentle" filter with a slope of, say two decibels per octave. For high end audio enthusiasts.

Or a "violent" filter which easily provides stopband slopes of *hundreds of decibels* per octave. Hitting a brick wall. With both "perfect" tuning and "distortoinless" phasing.

At any rate, a lot of you asked for extra details on a "real" digital filter design. Finding magic coefficients the way the pros do. I guess we can attack this in a future column. But first, we will need to explore certain fundamental background math. Math that involves…

Linear Equations

The key to any digital filter design is finding *coefficients*. Hopefully, you find what you want in a book or ap note. If not, you start with a pile of equations and stuff the amplitudes and frequencies you desire in them. Then you solve the equations to find your coefficients. The tricky part is that you will typically be solving for *lots* of frequencies and amplitudes.

The math usually involves *linear equations*. Ferinstance…

$$x - 3y = 9$$
$$x + 2y = -1$$

This is one example of a pair of *linear algebraic equations*. We have two variables in two unknowns. Any of several methods can be used to find out that $x = 3$ and $y = -2$. You might subtract the second from the first, leaving one equation in y. Or solve the first for x and substitute its value into the second.

The system is linear because the only power of x or y involved is the first. When the number of equations equals the number of unknowns, you can usually find all the unknowns.

Often, you'd expect *one* solution for x and y here. But in special cases, there can be *zero* or *many* solutions.

You evaluate these oddball cases by graphing your equation as a pair of straight lines. Should the lines cross, you have one solution.

If the lines are parallel, you have no solution. And if the lines extend into one longer line, you will have an infinite number of solutions.

In a digital filter, you might want to solve ten linear equations for ten amplitudes at ten frequencies. While you can still solve your first equation for one variable, plug this into the second, plug that into the third, and so on, this *Gausian elimination* gets ugly in a big hurry.

Instead, we go to a super elegant (though obscure and awful sounding) process that involves…

Determinants

Often when mathematicians face a messy number problem, they'll start working with entire piles of numbers instead. A pile of numbers is called a *matrix*. A matrix arranged into rows and columns is *two dimensional*. And those having an identical number of rows and columns are *square*.

Do remember that columns support buildings; thus they go up and down. The rows always go across a stadium. Back and forth.

A *matrix operator* takes one pile of numbers and generates a new pile of numbers from it. Following some rule or set of rules. Matrix operators are often dumb-witted, orderly, and very repetitive.

One extremely important matrix operator is known as a *determinant*.

A pile of numbers is called a *matrix*. A matrix is *two dimensional* if it can be arranged into rows and columns. A matrix is *square* if the number of rows equal the number of columns. Math that takes a pile of numbers and creates a new number or pile of numbers is called a *matrix operator*.

One popular matrix operator is called a *determinant*. A determinant takes a pile of numbers and calculates *one* new number from that pile.

The determinant for a 2X2 matrix is the difference of the cross product of its diagonals…

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Higher order determinants are found by *reducing* them. A selected row and column is crossed out, leaving a *subdeterminant*. The value at the crossout is multiplied by the subdeterminant. Note that signs alternate across the row as the products are summed. Here is how a 3X3 is reduced…

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

The process continues for higher orders. Such as this 4X4…

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = a \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

Fig. 1 – DETERMINANTS ARE A POWERFUL METHOD for solving complex linear equations. Such as those needed for digital filter designs.

Consider this set of four linear equations in four unknowns…

$$3w - 2x + 1y + 4z = -9$$
$$4w + 4x - 6y + 9z = 1$$
$$2w - 2x + 1y + 4z = 6$$
$$5w + 4x - 3y - 7z = -3$$

Make a 4x4 *system determinant* out of the numbers in front of the variables. Then make four *variable determinants* by selectively *substituting* the right side result in each *w, x, y,* or *z* column. Then divide variable by system…

$$w = \frac{\begin{vmatrix} -9 & -2 & 1 & 4 \\ 1 & 4 & -6 & 9 \\ 6 & -2 & 1 & 4 \\ -3 & 4 & -3 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 & 4 \\ 4 & 4 & -6 & 9 \\ 2 & -2 & 1 & 4 \\ 5 & 4 & -3 & -7 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} 3 & -9 & 1 & 4 \\ 4 & 1 & -6 & 9 \\ 2 & 6 & 1 & 4 \\ 5 & -3 & -3 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 & 4 \\ 4 & 4 & -6 & 9 \\ 2 & -2 & 1 & 4 \\ 5 & 4 & -3 & -7 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} 3 & -2 & -9 & 4 \\ 4 & 4 & 1 & 9 \\ 2 & -2 & 6 & 4 \\ 5 & 4 & -3 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 & 4 \\ 4 & 4 & -6 & 9 \\ 2 & -2 & 1 & 4 \\ 5 & 4 & -3 & -7 \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} 3 & -2 & 1 & -9 \\ 4 & 4 & -6 & 1 \\ 2 & -2 & 1 & 6 \\ 5 & 4 & -3 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 & 4 \\ 4 & 4 & -6 & 9 \\ 2 & -2 & 1 & 4 \\ 5 & 4 & -3 & -7 \end{vmatrix}}$$

Fig. 2 – HOW DETERMINANTS CAN BE USED to solve linear equations.

This one does multiplies and adds to a square matrix and then generates a single numeric value.

Determinants happen to excel at ridiculously simplifying the solution of high order linear equations.

Consider this 2x2 square matrix…

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Its determinant is found when you multiply the diagonals together and subtract them…

$$D = ad - bc$$

A determinant for a larger square matrix is found per all the details in figure one. A 3x3 determinant…

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

Will evaluate thusly…

$$D = a(ei\text{-}fh) - b(di\text{-}fg) + c(dh\text{-}eg)$$

The game can continue for larger determinants. Find a value in some row and column. Maybe the top row. Find the *subdeterminant* that is left when you *strike out* the *row* and the *column* of that value. Evaluate your subdeterminant and multiply by that value. Be sure to *alternate the sign when you continue across (or down) your chosen row or column.*

A determinant is just a series of multiplies and adds. Which accept a square matrix and *determine* a single numeric from rows and columns.

Now for the neat part. Figure two shows us how determinants can solve linear equations. Such as …

$$Ax + By + Cz = J$$
$$Dx + Ey + Fz = K$$
$$Gx + Hy + Iz = L$$

Assume we do know constants A through L and want to find unknowns *x* through *z*. The *system determinant* is found using elements A through I. Your *x* determinant can be found by *substituting* for J, K, and L in the *x* column positions.

Your *y* determinant similarly gets found by substituting J, K, and L in the *y* column. Same goes for *z*.

And our messy linear equations simply become…

$$x = (x\text{det})/(\text{sysdet})$$
$$y = (y\text{det})/(\text{sysdet})$$
$$z = (z\text{det})/(\text{sysdet})$$

All you are doing here is simple multiplies and adds. Followed by a single division. For each variable. Whenever the system determinant is zero, the equations blow up, and you have no solutions at all.

An indeterminate determinate.

Additional details on determinants can be located in most any advanced algebra textbook.

faster! FASTER!

With PostScript, you can easily do an 8x8 determinant in 70 milliseconds or less. Directly solving an 8x8 linear equation will take *nine* times as long. One determinant used as the system denominator; eight for the variables.

Call the 8x8 determinant time *t*. If you blindly go to a 9x9 solution, you will need 9*t* for the determinant. And 10x9 = 90*t* for the full solution. Thus, the raw 9x9 would take 10 times as long. A 10x10 would take 10 x 11 = 110 times as long, and so on.

Which gets way out of hand.

So, we'll think smarter instead of harder. Figure three shows us some handy speedup tricks. These get real important above 8x8.

First and foremost, use a system that does not take forever to do some floating point multiplies. Any newer microprocessor, any DSP, or a math coprocessor should help bunches.

Second, brute force linear code your 4x4. Instead of using four 3x3's that call twelve 2x2's. This reduces the overhead on your innermost (and most often called) service loop.

Third is the real biggie. There's a sneaky and off-the-wall rule that tells us: *A determinant stays unchanged if you replace the value in a row with itself added to a constant multiplied by the value in any other row.*

So, use this oddball rule to *force zeros.* For there is no point at all in solving any subdeterminant if you are going to multiply it by zero in the next step. Thus, if you *force zeros first,* an 8x8 determinant only has to calculate *one* 7x7, not eight of them.

How many mathematicians does it take to change a light bulb?

One, who hands the bulb to five Californians. Reducing the problem to a previously solved riddle.

Similarly, we know a full 9x9 is going to take *much* longer than an 8x8. So why not use the 9x9 to solve for only *one* variable?

Then substitute this one variable value *back* into the eight remaining equations. Then solve a faster 8x8.

This last trick lets you find a 9x9 in 1.2 times that of an 8x8.

Substituting a "compound interest" for a "factorial blowup". You can do a PostScript 10x10 in 2.1 seconds.

The speedup details can be a tad system specific. You do have to take into account system overhead versus multiply times and so on. And a few new divisions may be needed.

Thus, linear coding a 5x5 or zero forcing a 6x6 may or may not help you much. And could actually slow you down. But those are all details; the speedup tricks certainly help a lot for 8x8 or higher problems.

In a real big hurry.

I've posted my PostScript code for all this as file LINEAREQ.PS to my www.tinaja.com

Induction Heating

I've sure gotten a lot of calls over this topic. This is something of an arcane backwater. A long time ago, someone noticed that tranformer and coil cores sometimes got painfully warm. They then decided to try and do this on purpose.

An *induction heater* places a fairly conductive object inside a resonant coil. Radio frequency energy routed through the coil gets transferred by induction to the object. Heating it.

A non-magnetic conductor heats by way of the *eddy effect*. Magnetic conductors heat through an additional *hysteresis loss*. Either way, energy is mostly transfered by the *magnetic*, or the "H" field.

Most induction heaters are really nothing but a transformer having a purposely shorted turn.

Induction heating is most popular with iron or steel objects. But nearly any lossy but otherwise fairly strong conductor can work.

Somewhat related are *dielectric* heaters. A dielectric heater places an

1. Avoid long floating point multiply times. Use a newer microprocessor, a DSP chip, or consider adding a math coprocessor.

2. Brute force your 4X4 code. This can reduce the overhead on your critical innermost timing loop.

3. Force zeros in all but one top row entry by using an obscure rule that says *"A determinant remains unchanged if any row is added to another row multiplied by a constant"*. There is no need to evaluate a subdeterminant if it is only going to get multiplied by zero in the next step. Like so…

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a+Kg & b+Kh & c+Ki \\ d & e & f \\ g & h & i \end{vmatrix}$$

First, let constant $K1 = -b/h$ so that $b+Kh = 0$. Substitute these new values into your determinant. Then let a new constant $K2 = -c/i$. Substitute again. This leaves you with zeros in the "b" and "c" column positions. Thus, only the "a" subdeterminant needs evaluated. Speedup is substantial.

4. Solve the highest order equations for only *one* variable. Then substitute this into a lesser solution. For instance, suppose we want to solve…

$$\begin{aligned} aw + bx + cy + dz &= J \\ ew + fx + gy + hz &= K \\ iw + jx + ky + lz &= L \\ mw + nx + oy + pz &= M \end{aligned}$$

Solve this 4X4 linear equation *only* for *w*. Since variable *w* is now known, we can call it *constant* W. And combine it with constants J through M. Leaving this much faster solving set of 3X3 equations…

$$\begin{aligned} fx + gy + hz &= K\text{-}We \\ jx + \qquad lz &= L\text{-}Wi \\ nx + ky + \qquad &= M\text{-}Wm \\ oy \qquad pz \end{aligned}$$

Fig. 3 – COMPUTATION TIMES GET RIDICULOUSLY out of hand when you use "raw" determinants to solve linear equations above 8x8. Here are some sneaky speedup tricks that I used in my LINEAREQ.PS utility.
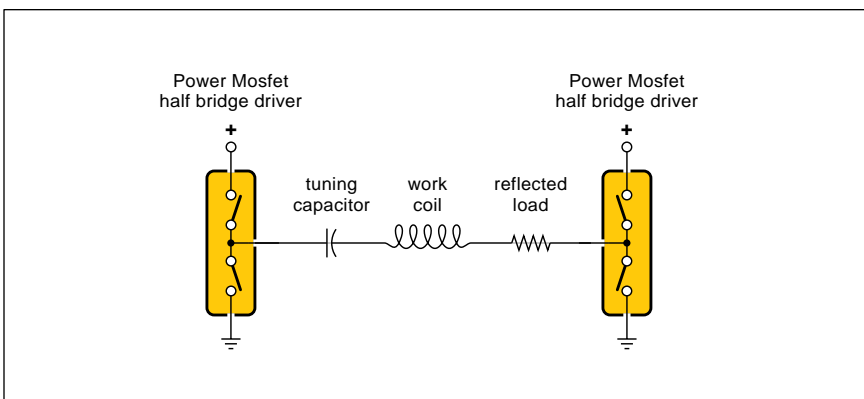

Fig. 5 – SIMPLIFIED SCHEMATIC for an induction heater. Diagonally opposite switches are alternately closed, creating a square wave drive at the heater's resonant frequency. Current mode drive circuits are also possible.

object which is only somewhat lossy between the plates of a resonanting capacitor. Radio frequency energy is transferred by dielectric coupling to the object. Energy is transfered by the *electric* or "E" field.

Dielectric heating works best with glues. Or similar lossy but otherwise fairly good *insulators*. Uh, except for food, dielectric heating is much less common than induction heating.

Those conductive capacitor plates can also optionally apply pressure.

A microwave oven can be both an induction heater and dielectric heater. The target is put inside a waveguide or cavity. Energy can be transferred using either or both fields.

Unfortunately, the Maggie in the oven blows up when you try actually using the induction part of the field by putting any metal inside. Because of mismatch and reflected power.

I've shown three radio frequency heating approaches in figure four.

The advantages of induction or dielectric heating are that you'll heat *only* the object. Not the environment. You exactly control the temperature and its rate of heating or cooling. As well as the heating depth profile and the total energy transferred.

This (usually) contactless process avoids contamination. And can even be done inside a vacuum.

A few important uses for induction heaters often include heat treating of metals, brazing or soldering, and for shrink fit assembling.

Thanks to the frequency sensitive *skin effect*, you can carefully control the depth of heat penetration. Thus, induction is ideally suited for *surface hardening* of running parts. Such as crankshafts or CV universal joints.

Some of the new "cool top" stove designs also apply induction heating. Dielectric heating gets used to make plywood, for materials research, and in various medical treatments.

And for warming your lunch.

The FCC, of course, regulates all induction and dielectric heaters. First with everything else in part 15. And then as industrial electronics in their short and obscure part 18. One source for these regs are the *US Government Bookstores* in many cities.

I was unable to find any of the less common FCC regs on line.

A popular operating frequency was 27.12 MegaHertz with a tolerance of 326 kiloHertz. Please do note that the harmonics of an improperly shielded induction heater at this frequency might totally trash television channel two and much of the FM band.

The newer induction heaters often operate in the 50-200 kiloHertz range. Your chosen frequency gets carefully optimized for the size of the load and the depth of penetration desired. But even raw 60 Hertz ac power can get used to heat massive objects. Such as aluminium billets.

Earlier induction heaters required humongous vacuum tube transmitters or motor generator sets. These days, an induction heater is a variation on a switchmode power supply.

Where power FET's or whatever square wave drive a lossy but series resonant load. Parallel current modes are also possible.

Figure five shows us a simplified induction heater schematic.

Way on back in high school, I was involved in an early induction heater. It was just a tube style ham 11 meter transmitter of a few hundred watts. Driving a largish load coil. Inside the coil was this highly temp resistant glass tube. Place a large nail in the tube and it would light up a bright cherry red within a few seconds.

You could also cook hot dogs, but you had to roll the hot dogs in iron

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NAMES AND NUMBERS

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filings first. We finally opted for a central coat hanger wire. Even then, it usually burned the middle and left the outside cold.

The dangers inherent in home lab induction heat experiments are too numerous to mention here. I'll save them as an exercise for the student.

Yeah, its gonna be on the test.

Getting Info

Induction heating system design is tricky. Your frequency, power level, and your coils have to get carefully matched to the objects being heated. And to the reasons for the heating in the first place. Tuning is critical.

It is fairly easy to find useful info on induction heating. Once you dig around in obscure enough places.

Starting with the web.

I've just added a link for a super searching service to *www.tinaja.com* that *simultaneously* lets you access *many dozens* of the more popular major search engines.

I've gathered some resources for you into this month's sidebar.

The best books seems to be the two volumes of the oldie but goodie *Basics of Induction Heating* by one Chester Tudbury. The only source for this text I have been able to find is through *InductoHeat*.

The better volume one concerns itself with coil and load fundamentals and heating calculations. Volume two is on 1960 transmitter circuit designs, tuning, and matching.

A 1988 book is titled *Elements of Induction Heating*. From *ASM Press*.

I also found a *Franklin* reprint about the 1969 *Industrial Applications of Induction Heating* text.

There is also the specialized *High Frequency Plasma Heating* from the *American Institute of Physics*. Have not seen this one, though.

I was unable to find any specific trade journal or association. But try the *IEEE Transactions on Industrial Applications* or their *Transactions on Industrial Electronics*.

For heating apps in general, check into *Industrial Heating* or *Process Heating*. For the new cool stovetops, check into *Appliance* and *Appliance Manufacturer*.

For power electronics, *PCIM*.

There are seminars. *InductoHeat* is one source. In particular, check out their two-volume proceedings from the *Sixth Annual Induction Heating Seminar*. Two thick self-published volumes cover heat treating and mass heating. Having lots of solid design details. Dozens of free ap notes are also newly offered.

The *ASM* also promotes induction heating seminars and courses.

One high profile induction heating supplier is *InductoHeat*. Competitors include *Tocco*, *Fuji*, *Huttinger*, and *American Induction Heating*. Who are big on forgery.

Useful sources for coils and flux concentrators include *Alphaform* and *Lepel*. Lepel is part of *InductoHeat*.

And *Raychem* is big on induction heated shrink tubing.

One used induction heater buy-sell service is *Tocco*. But more brands are

(A) With an induction heater, energy is inductively coupled to a fairly conductive load. Energy is linked by the magnetic "H" field.

(B) With a dielectric heater, energy is electrically coupled to a fairly non-conductive load. Energy is linked by the electric "E" field.

(C) In a microwave oven, energy is coupled by both the "E" and the "H" fields. Depending upon load conductivity and position.
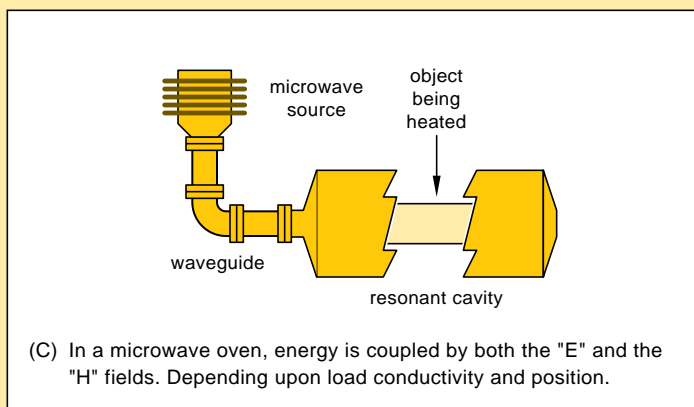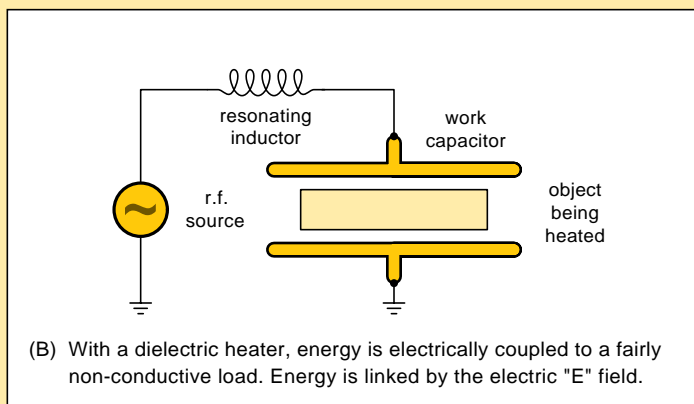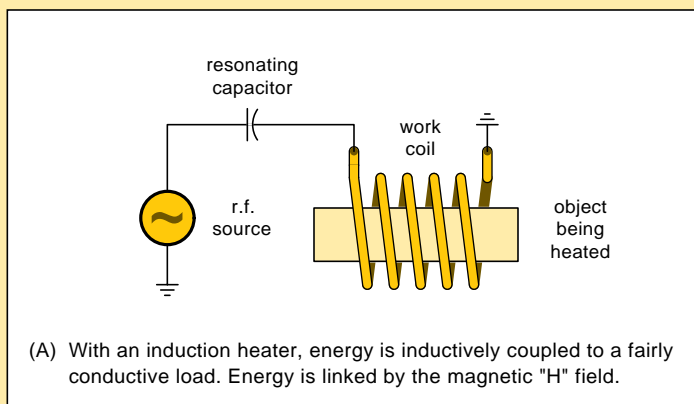
Fig. 4 – THREE ROUTES towards radio frequency heating.

found in the *Used Equipment Digest* or *Surplus World*. On your own, you could probably pick up a reworkable ham or mil surplus transmitter at *Fair Radio Sales*. Better still, modify a high level switch mode power supply or PWM motor driver.

Book-on-demand Binding

The *Unibind* people just came up with an interesting new home binding solution. Its called the *Binding Spine*, and is basically a "U" shaped plastic channel with some hot glue in it. Plus a temporary positioning tag.

You pop your text and your front and back covers in it and drop it in a Unibind toaster. Results look fairly sharp. Cost is under a dollar.

Three gotchas: You have to adjust your content to fit one of the fixed sizes. In eighth inch increments. And

you can not easily letter your spine. And it fails the "Turing test". Where it looks wildly different from a "real" binding from a "real" printer.

Nonetheless, a new home binding option is certainly welcome. I still like Unibind's *Pentabind* best of all. But all we really require is a sanely priced prescored sheet of self-stick hot glue on a carrier. *Planax* comes close, except for high prices.

There is no reason such a product cannot be offered at seven cents per binding. User retail. And the new H-P 5SiMX makes full wraparound covers trivial. So, there are some great new oppoknockitys tuning here.

Contact Ron Leonard at *Unibind* for more details.

New Tech Lit

Electric power generators reaching a stunning 60 percent efficiency are described in the July, 1966 issue of *Power Engineering*. The big trick is to start off with a gas turbine, put a steam generator in the middle, and bottom out with an ammonia cycle.

Wood Machining News is a pricey newsletter on precision sawing. For cabinet makers and such. And that *Maher Messenger* is a ventriloquism catalog and newsletter.

Who said that?

Free samples of *Nyliner* polymer bearings are offered by *Thomson*.

Inventor assistance source lists from R.L. Conger of *Battelle Pacific Northwest*. The first is intended for non-commercial help. And a second for commercial services.

But, these days, only a *total idiot* would purposely call themselves an inventor. To do so is precisely the same as wearing a big animated neon talking sign that says *"Please rip me off"*. Much more on this in my *Case Against Patents* package.

More on BOD binding options and opportunities in my *Book-on-demand* publishing kit. Found per my nearby *Synergetics* ad. And a reminder to be sure to visit my new web site you'll find at *www.tinaja.com*

As usual, most of the mentioned items appear either in the *Names & Numbers* or the *Induction Heating Resources* sidebars. Be sure to check there before calling our no charge technical helpline listed in the *Need Help?* box. ◆