The recursive midpoint rule for curve-drawing was discovered in 1959 by Paul de Casteljau, who showed that the curve could be described algebraically by the remarkably simple formula

$$z\left(t\right) = \left(1 - t\right)^3 \, z_0 \, + \, 3\left(1 - t\right)^2 \, z_1 \, + \, 3\left(1 - t\right)t^2 \, z_2 \, + \, t^3 \, z_3\,,$$

as the parameter t varies from 0 to 1. This polynomial of degree 3 in t is called a *Bernshteĭn polynomial*, because Sergeĭ N. Bernshteĭn introduced such functions in 1912 as part of his pioneering work on approximation theory. Curves traced out by Bershteĭn polynomials of degree 3 are often called *Bézier cubics*, after Pierre Bézier who realized their importance for computer-aided design during the 1960s.

Here is a graphic example of the recursive midpoint rule.