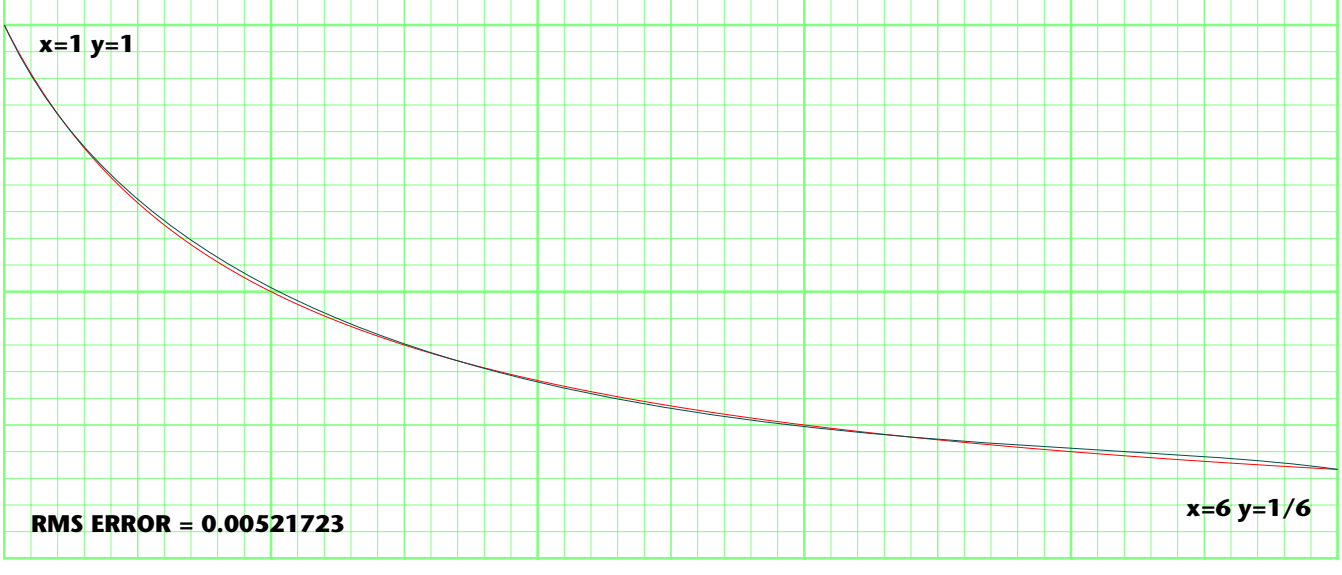
HYPERBOLIC CUBIC SPLINE CURVE FITTING

By Don Lancaster

A consulting client of mine has asked for an exceptionally precise variant on a cubic spline approximation to the **y=1/x** hyperbola over the range of **x=1 to x=6**. How many splines are needed for acceptible accuracy?

The basic 1/x hyperbola is plotted in red below, with 10X horizontal and 20x vertical emphasis. A single spline five point fit was done and then optimized for minimum rms error. Optimization was done by incremental adjustment of enty and exit angle and distance to the first and second influence points. The spline appears as aqua below.

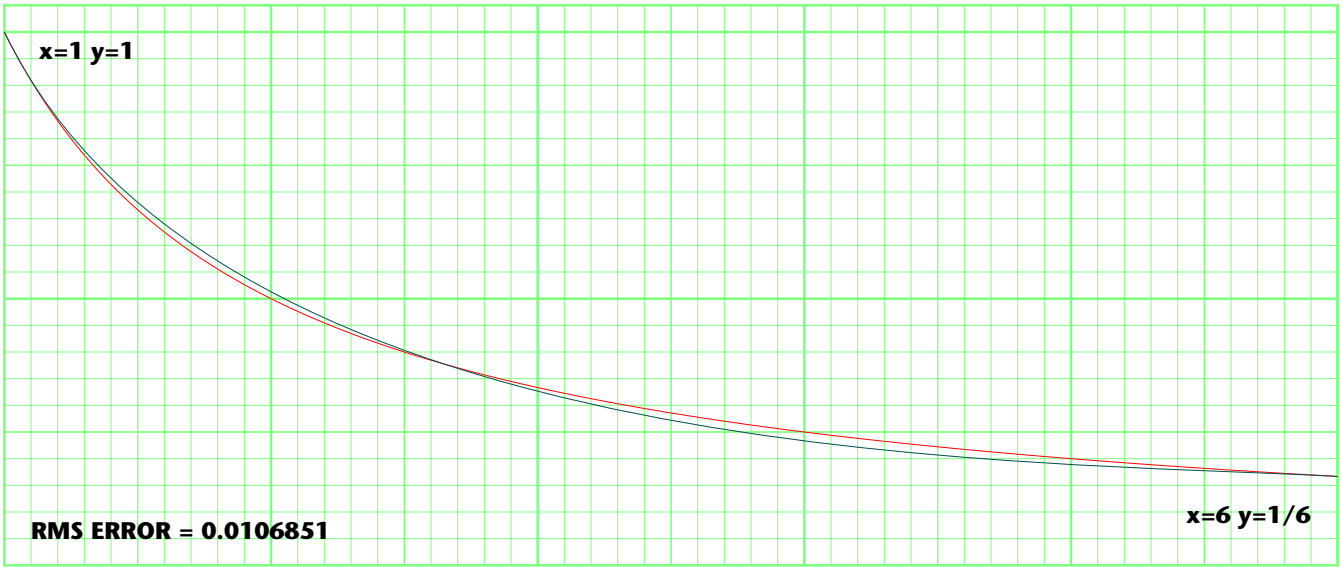
Although the overall fit is quite good, the rising end response and non-monotonicity at near x=6 is unacceptable for an application involving camera resolution charts. Use maximum magnification in Acrobat to view these differences.



The rising response and the apparent lack of monotonicity can be eliminated by forcing the ending slope at x=6 to exactly match its expected value of 1/36.

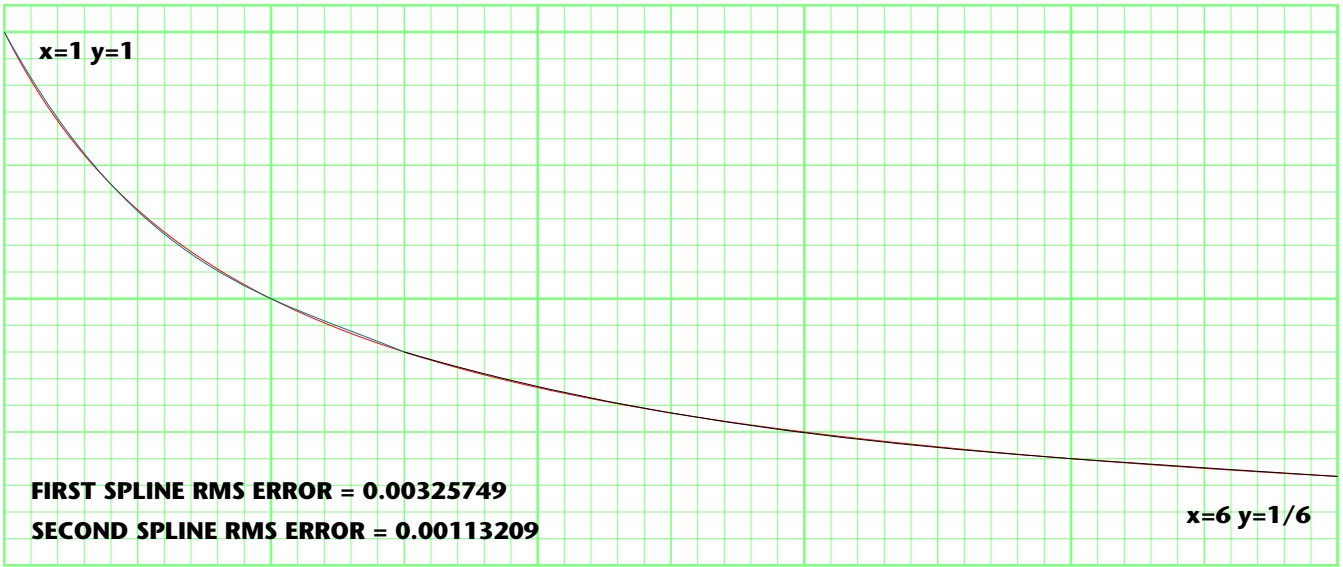
This new constraint can only make the overall rms error worse, but at least the errors are in less demanding portions of the curve.

The result seems even more unacceptable, suggesting a minimum of two splines will be needed for the approximation.



A stock Jim Fitzsimons **BEZ4PTS** utility can fit a pair of splines to exactly match seven data points as shown here.

Slight errors do remain and a visible slope discontinuity can be observed. But the fast and convenient results are probably "good enough" for the application at hand.



A custom pair of splines can further improve the results by forcing slope continuity at their joint and by floating the seven zero crossings for optimum results as shown here. Considerably additional time and effort is involved.

A two spline fit can further be improved by matching high and low spline rms errors and by floating the initial and final angles. These are left as an exercise for the serious student.