

The problem is to connect two Bézier curves.

This is a Bézier curve.

$$x = ((a t + b) t + c) t + d$$

$$y = ((e t + f) t + g) t + h$$

This is the first Bézier curve. $x = x_0$ when $t = 0$.

$$x_0 = ((a_0 0 + b_0) 0 + c_0) 0 + d_0$$

Solve for d_0 .

$$d_0 = x_0$$

Replace d_0 .

$$x = ((a_0 t + b_0) t + c_0) t + x_0$$

$$x - x_0 = a_0 t^3 + b_0 t^2 + c_0 t$$

$x = x_3$ when $t = 1$.

$$x_3 - x_0 = a_0 1^3 + b_0 1^2 + c_0 1$$

Solve for c_0 .

$$c_0 = -a_0 - b_0 - x_0 + x_3$$

Replace c_0 .

$$x - x_0 = a_0 t^3 + b_0 t^2 + (-a_0 - b_0 - x_0 + x_3) t$$

$$x - x_0 = t (a_0 (t^2 - 1) + b_0 (t - 1) - x_0 + x_3)$$

Find the slope.

$$\frac{d}{dt} (x - x_0 = t (a_0 (t^2 - 1) + b_0 (t - 1) - x_0 + x_3))$$

$$\frac{dx}{dt} = a_0 (3 t^2 - 1) + b_0 (2 t - 1) - x_0 + x_3$$

Define the slope.

$$q = \frac{dx}{dt}$$

$$q = a_0 (3 t^2 - 1) + b_0 (2 t - 1) - x_0 + x_3$$

$q = q_0$ when $t = 0$. Solve in graph space.

$$q_0 = \frac{x_1 - x_0}{\left(\frac{1}{3}\right)}$$

$$q_0 = 3(x_1 - x_0)$$

Solve in equation space.

$$q_0 = a_0 (3 * 0^2 - 1) + b_0 (2 * 0 - 1) - x_0 + x_3$$

$$q_0 = -a_0 - b_0 - x_0 + x_3$$

(1)

$q = q_3$ when $t = 1$. Solve in graph space.

$$q_3 = \frac{x_3 - x_2}{\left(\frac{1}{3}\right)}$$

$$q_3 = 3(x_3 - x_2)$$

Solve in equation space.

$$q_3 = a_0 (3 * 1^2 - 1) + b_0 (2 * 1 - 1) - x_0 + x_3$$

$$q_3 = 2 a_0 + b_0 - x_0 + x_3$$

(2)

Add equation (1) and equation (2).

$$q_0 + q_3 = a_0 - 2x_0 + 2x_3$$

Solve for a_0 .

$$a_0 = q_0 + q_3 + 2(x_0 - x_3)$$

Solve for b_0 .

$$b_0 = -a_0 - q_0 - x_0 + x_3$$

$$b_0 = -(q_0 + q_3 + 2(x_0 - x_3)) - q_0 - x_0 + x_3$$

$$b_0 = -2q_0 - q_3 - 3x_0 + 3x_3$$

Find the rate of change of the slope.

$$\frac{d}{dt}(q = a_0(3t^2 - 1) + b_0(2t - 1) - x_0 + x_3)$$

$$\frac{dq}{dt} = 6a_0t + 2b_0$$

Define the rate of change of the slope.

$$p = \frac{dq}{dt}$$

$$p = 6a_0t + 2b_0$$

$p = p_0$ when $t = 0$.

$$p_0 = 6a_0 \cdot 0 + 2b_0$$

$$p_0 = 2b_0$$

$p = p_3$ when $t = 1$.

$$p_3 = 6a_0 \cdot 1 + 2b_0$$

$$p_3 = 6a_0 + 2b_0$$

$$p_3 = 6a_0 + p_0$$

Solve for a_0 .

$$a_0 = \frac{p_3 - p_0}{6}$$

This is the second Bezier curve. It has the same slope and the same rate of change of slope at the common point $[x_3, y_3]$. The other end is the point $[x_6, y_6]$.

$$x - x_3 = t(a_1(t^2 - 1) + b_1(t - 1) - x_3 + x_6)$$

$$q = a_1(3t^2 - 1) + b_1(2t - 1) - x_3 + x_6$$

$$p = 6a_1t + 2b_1$$

$$q_3 = a_1(3 \cdot 0^2 - 1) + b_1(2 \cdot 0 - 1) - x_3 + x_6$$

$$q_3 = -a_1 - b_1 - x_3 + x_6$$

$$q_6 = a_1(3 \cdot 1^2 - 1) + b_1(2 \cdot 1 - 1) - x_3 + x_6$$

$$q_6 = 2a_1 + b_1 - x_3 + x_6$$

$$p_3 = 6a_1 \cdot 0 + 2b_1$$

$$p_3 = 2b_1$$

$$p_6 = 6a_1 \cdot 1 + 2b_1$$

$$p_6 = 6 a_1 + 2 b_1$$

The same equations for the first curve are:

$$q_0 = -a_0 - b_0 - x_0 + x_3$$

$$q_3 = 2 a_0 + b_0 - x_0 + x_3$$

$$p_0 = 2 b_0$$

$$p_3 = 6 a_0 + 2 b_0$$

Solve the eight equations.

$$a_0 = -\frac{5 p_0 + p_6 - 6 (x_0 - 2 x_3 + x_6)}{24}$$

$$a_1 = \frac{p_0 + 5 p_6 - 6 (x_0 - 2 x_3 + x_6)}{24}$$

$$b_0 = \frac{p_0}{2}$$

$$b_1 = -\frac{p_0 + p_6 - 6 (x_0 - 2 x_3 + x_6)}{8}$$

$$p_3 = -\frac{p_0 + p_6 - 6 (x_0 - 2 x_3 + x_6)}{4}$$

$$q_3 = \frac{p_0 - p_6 + 6 (x_6 - x_0)}{12}$$

$$q_0 = -\frac{7 p_0 - p_6 + 6 (5 x_0 - 6 x_3 + x_6)}{24}$$

$$q_6 = -\frac{p_0 - 7 p_6 - 6 (x_0 - 6 x_3 + 5 x_6)}{24}$$