THE GURU'S LAIR

Don Lancaster

The Quest for Magic Sinewaves

Upping Power Electronics Efficiency

> Table lookup of long binary sequences offers compelling advantages over PWM. All you have to do is exhaustively explore all possible 420 bit words. Using PostScript, of course.

There is a bunch of fresh interest these days in higher power digital sinewaves. For everything from induction motor speed controls to electric autos to UPS power quality to phone ringers to off-grid solar inverters.

As figure one shows us, the key to all of these aps is to start with a dc supply and some switches. You then digitally flip these switches in some sequence to try and produce a clean power sinewave in your motor or transformer winding.

This switching arrangement is usually known as an *H bridge drive*. With switches in positions B and D or in A and C, there will be zero new motor or transformer current. When in positions A and D, *positive* new motor current will get added to the present waveform. But in positions B and C *negative* motor current gets removed instead.

Obvious goals here are to provide a variable frequency and amplitude. Plus low harmonics. With a zero dc term. To maximize efficiency, we'll want to use as *few* switch flips as possible per cycle. Plus, of course, we want end up purely digital and microcontroller friendly. The main question is "Can we find some magic new digital sinewave switching ploy that meets these goals?"

The old-line stock solution here was once known as...

PULSE WIDTH MODULATION

Or PWM for short. With PWM, you start by using a high frequency carrier. Say 20 kiloHertz for 60 Hertz power. As with most any FM or PM scheme, your *duty cycle* gets varied. By averaging, or *integrating* out the carrier's duty cycle, a low frequency modulation can be recovered. This integration normally gets done when the inductance of the motor winding acts as a low pass filter.

The big attraction to PWM is the ease with which both amplitude and frequency can be changed. An analog PWM can be easily generated using a sawtooth and a comparator.

But there are a few grievous flaws to PWM. There are inherently a lot of switch flips per cycle. Each flip will end up costing you dearly with high frequency losses. Losses mean higher temperatures, more expensive drive transistors, and larger heatsinks.

PWM carrier amplitude is *always* larger than the fundamental.

Worse, each transition in stock PWM is typically a *double* flip. You change *both* sides of your bridge at once. Giving you an additional 2X efficiency penalty. The number of switch flips for each cycle usually is totally independent of your output amplitude. So low amplitudes mean lousy efficiencies. There is always substantial energy kicking around at unwanted high frequencies. Which can lead to whine or noise.

While all of the low harmonics can theoretically be eliminated, the real world may not work that way. Any noise, distortion, quantization, nonlinearities, or a dc term in your PWM modulation will directly show up as output imperfections.

USING "MAGIC" SINEWAVES

Lately, I've been exploring a new *magic sinewave* approach that seems to offer many advantages over stock PWM. Not to mention being utterly fascinating and highly addictive.

Take a big long string of ones and

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zeros. When repeated, this string will possess a *Fourier Series* consisting of a fundamental and some harmonics. By picking all of your ones and zeros precisely right, we can force most of the lower harmonics to zero. *And still provide a variable amplitude output*. Just like PWM.

This ploy is very microcontroller friendly. For amplitude 78, you look up sequence 78 in a table and then send it. For frequency, you adjust the dwell time between lookups.

There is no high frequency PWM carrier. All those switch flips can be dramatically minimized. And there's no modulation or integration hassles. Besides, no noise or distortion-prone analog sawtooth is involved.

So far, I have found that the most interesting results use word lengths of 210 and 420 bits. Highly addictive fascination comes about when you try to find useful methods to work around exhaustively searching all of those possible 420 bit words.

SHORTER SOLUTIONS

Let us start with the simpler and shorter sequences of figure 2.

My personal preference here is to use the general purpose *PostScript* computer language. Mostly because it is so incredibly friendly, intuitive, and serendipitous. Plus being freely available, as a *GhostScript* clone.

I've written a simple PostScript FOURIER.PS analyzer. This has been recently posted both to my personal *www.tinaja.com* web site and to the *Circuit Cellar* site. It will analyze any sequential string of ones and zeros. And then give you the size of the fundamental and any harmonics of interest. In several formats. With or without selective oversampling.

Let's see. For most any sequence length, we can guarantee a zero dc term by having an equal number of +1 and -1 switch states. This is often important to eliminate any level bias or iron saturation effects.

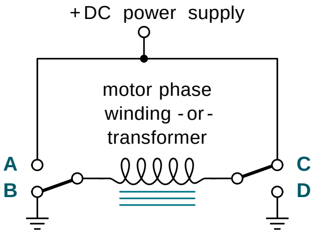
We can force all even harmonics to zero by providing for a *half wave symmetry*. Since your second is by far your most serious problem, you will usually force zero evens.

For half wave symmetry, you'll mirror top to bottom but *not* left to right. Ferinstance, on a 30 bit word, when bit (n) is a zero, bit (n+15) must also be a zero. If bit (m) is a +1, then bit (m+15) must be a -1.

Your obvious starting point is trying a square wave. Three major gotchas with this dude, though. The first and worst is that there is only one nonzero fundamental amplitude available. A second is that the third harmonic is a horrendous 33% and your fifth is a largish 20% of your fundamental. Finally, since you are always flipping from +1 to -1, you will include an efficiency-robbing double transition.

Still, a square wave is cheap and simple. When you can live with fixed amplitude and if strong harmonics are no problem, go for it.

Solutions that are *trinary* and



Switches get flipped in the proper sequence to synthesize a sinwave current of the desired amplitude, phasing, and frequency.

In position **A-D**, a positive current gets added. In position **B-C**, a negative current gets removed. Positions **B-D** or **A-C** neither add nor remove any current.

Figure 1 – *How power sinewaves can be created from a dc supply.*

Your obvious starting point is a plain old square wave. This will cancel all even harmonics, but has a very strong third and fifth. It also gives you only a single fundamental amplitude and has a double transition…

A 12 bit word gives you four sequences for four fundamental amplitudes that have no third harmonics. One has a double transition…

There are nine 30 bit words having no third or fifth harmonic. They can give you nine different fundamental amplitudes. Two have double transitions…

Of the 2219 possible 210 bit words which have no third, fifth, seventh, or ninth harmonic, this one is often your best choice for fixed amplitude uses…

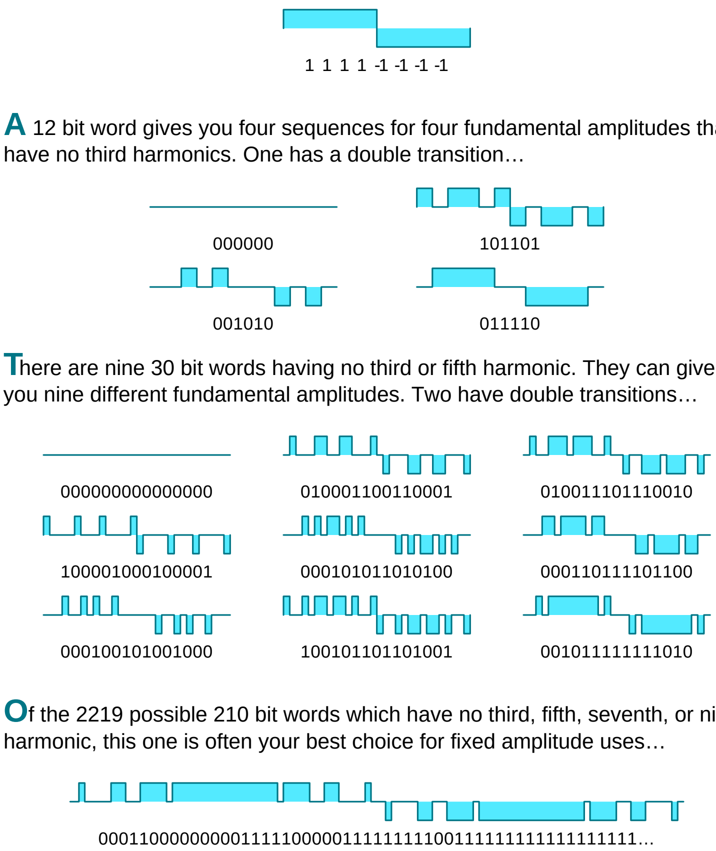


Figure 2 – *Exploring low harmonic binary sequences.*

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include allowable values of +1, -1, and 0 would appear to lead us up to cleaner and more desirable results. If we are careful to never have any +1 right beside a -1, then *all* the double switch flips can be avoided.

Figure 2 also shows us four 12-bit waveforms which completely cancel their third harmonics. Because of the equal number of +1 and -1 states, there is no dc term. And because of half wave symmetry, there are no even harmonics present.

The fundamental amplitudes of the four waveforms are 0%, 57%, 80% and 100%. On the strongest amplitude, the fifth harmonic is a tolerable 20% and the seventh is at 14%. Clearly we do have a much better solution here.

By using exhaustive searches or dumb luck, we can find sequences of ones and zeros which have low (but nonzero) values for any particular harmonic. These can sometimes be useful. But to force a truly zero *nth* harmonic, you'll have to go to bit lengths that are a *product* of *n* and some other numbers. Ferinstance, the only bit lengths that offer hope of *completely* cancelling out a third harmonic are 3, 6, 9, 12, 15,... The only lengths which can completely cancel a fifth are 5, 10, 15, 20, 25.... Thus, by exclusion, your only bit lengths that can completely cancel *both* the third and the fifth are 15, 30, 45, 60...

Figure 2 shows us nine 30-bit trinary waveforms that have no dc term, no even harmoics, zero third, and a zero fifth.

The strongest fundamental peak is 1.05 your supply voltage and has a 11.8% seventh and a zero ninth. You can pick amplitude levels of 0, 27, 43, 53, 65, 70, 80, 87, and 105 percent.

In many cases, the output power is more of a concern than the output voltage. These same ten waveforms give you relative powers of 0, 7, 17, 25, 38, 44, 58, 69 and 100 percent.

We now do have a way to adjust *both* the frequency and amplitude of our magic sinewave. But the total number of useful 30-bit amplitude steps is sorely limited. To beat this, we go to longer bit lengths.

A reasonable goal is to try and find *one hundred* amplitudes in one percent steps. This could meet most motor control needs without going to insanely long words or uselessly high frequencies. In figure 2, you can finally go to one super magic 210 bit waveform. 210 is the product of 2, 3, 5, and 7. So certain carefully selected sequences should cancel out. This particular waveshape has a zero dc, zero evens, zero third, fifth, seventh, fifteenth, and twenty-first. Your eleventh is an amazingly low 1.08 percent. And the thirteenth ain't half bad either. At 8.9 percent.

Yeah, bunches of very gruesome higher harmonics exist. After all, the square corners in your waveshape do have to evolve from *somewhere*. For using only a fundamental plus a pitifully weak eleventh certainly can't hack it by themselves. But very high harmonics are usually easy to filter. And none of them are any worse than the third on a plain old square wave.

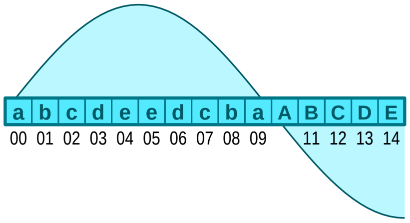
There's a mere *seven* transitions per quarter cycle for a total of 28!

It is interesting to compare this waveform against a 210-bit PWM waveshape. PWM might require 420 double transitions. Compared to 28 single transitions for the magic 210 sinewave. While we can't claim that our magic sinewave is *thirty times* more efficient, we certainly can say that the transition losses will often be thirty times worse with PWM. Or that we can get by with significantly smaller heatsinks and drivers.

One gotcha: When you mirror the quadrant, you will end up with 106 bits. Simply drop your final zero.

I have previously selected out a hundred useful 210-bit solutions. From the 2219 or so possible having zero third, fifth, or seventh. These have gotten posted as HACK87.PDF and related files. But I was not really

Say you want to find 60 bit words having a zero third harmonic. Use quarter wave symmetry to guarantee no even harmonics and no cosine terms. Draw out one-quarter cycle worth of third harmonic sine and label the bits as shown here...



Now, **a** will be the contribution by bit #00, determined by the sine of the angles at the start and end of the bit. And **A** will be minus this value. It does not matter what the value of **a** is. The only way you'll get a perfect cancellation is to make sure **a**, **a**, and **A** cancel each other out.

This can only happen if all three bits are zero or if there is only one **a** present to cancel out the value of **A**. For full cancellation, you can write these five equations...

00 + 09 = 10
01 + 08 = 11
02 + 07 = 12
03 + 06 = 14
04 + 05 = 15

Note further that only ones and zeros are acceptable in these equations. Since 00 and 09 cannot both simultaneously be one, we will need only consider three cases for each equation.

Out of the 32,768 possible quarter cycle words, there are only 3^5 = 243 quarter symmetric 60-bit sequences having a zero third harmonic.

Figure 3 – *How to force odd harmonics to zero.*

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happy with those excess numbers of transitions on some selections. Nor their amplitude uniformity or their distortions. Thus, our current magic sinewave quest here is to find some "mo betta" 420 bit words.

NEEDLES IN HAYSTACKS

The big problem with longer bit length words is that there are great heaping bunches of them. Even a 60 bit word has 1,152,921,504,606,846,976 states. So an exhaustive search won't hack it. Nor will any random or Monte Carlo selections.

So, we have to work smarter and not harder. We have already noted that half wave symmetry gets rid of even harmonics. It also slashes the total number of cases by chopping the analyzed bits in half.

To perform a traditional Fourier analysis, you find out how much of the waveshape can be absorbed by a harmonic sine and cosine term. To minimize your work load, it is often useful to force all your cosine terms to zero. To do this requires *quarter wave* symmetry. In which your left and right sides of each half waveform are mirror images of each other.

Thus, by using sine terms only, you again cut the number of bits in half. At the risk of loosing certain solutions. For a 420 bit result, we'll only need analyze 105 bits.

Uh, this still may take a while. Even with PostScript. So we'll need additional methods to dramatically reduce the candidate patterns. One trick is shown in figure 3. To cancel out the third harmonic, certain bit combinations must add up to zero. These can lead to a series of linear equations. To cancel the fifth, other combinations must add to zero. The same for a seventh. Solving all these equations together might very much reduce your search problem.

Figure 3 uses a 60 bit word as an example. There will be a fifteen bit quarter word. 32,768 states if we try an exhaustive search.

Can we further reduce this?

The third harmonic will have 3/4 cycles in the quarter word. Call your bits *abcdeedcbaABCDE*. With $-a = A$ and so on. Now, a will be some angle and will have some sine. Makes no difference what its value. Since all sines will end up different from each other, the only way you could get a perfect cancellation is if $a + a = A$.

Ferinstance, this says that bit 00 plus bit 09 must equal bit 10. Bit position one plus bit position eight will have to equal bit position eleven. And so on. Otherwise, we won't cancel.

Selected for maximum efficiency by allowing only 12 or fewer quadrant transitions.
Harmonics 0,2,3,4,5,6,8,9,10,12,14,15, and 16 are zero. Harmonic 7 is zero or low.
Only the first quadrant of each 420 bit word is shown…

```
00 - 000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
01 - 000000000000000000000000000000000000000000000000000000000010000000000000000000000000000000000000000000000
02 - 000000000000000000000000000000000000000000000000000000000000000000000000000000000000001000000000000000000
03 - 000000000000000000000000000000000000000000000001000000000000000000000000000000000000001000000000000000000
04 - 000000000000000000000000000000000000100000000000000000000000000010000000000000000000001000000000000000000
05 - 000000000000000000000000000000000000000010000000000000000000000000000000010000000000000000000000000010000
06 - 000000000000000000000000000000001000000000000000000000000010000000000000000000100000000000000001000000000
07 - 000000000000000000000000000010000000000000000000001000000000000000100000000000001000000000000100000000000
08 - 000000000000000000000000000000100000000000000000000000100000000000000000100000000000000100000000000000100
09 - 000000000000000000000000001000000000000000000001000000000000001000000000000100000000001000000000001000000

10 - 000110000000000000000000000000000000000000001100000000000000000000000000110000000000000000001100000000000
11 - 000000000000010000000000000000000000000001000000000001100000000000000000000000000011100000000000100000000
12 - 000000000000000000000000000000001100000000000000000000000000001100000000001100000000000000000000000000011
13 - 000000000000000000001000000000000000010000000000001000000000100000000100000001000000010000000100000100000
14 - 000000000000000000000010000000000000001000000000000010000000001000000000100000001000000010000000100000010
15 - 000000000000000000001000000000000000100000000001000000000100000001000000010000001000000100000010000001000
16 - 000000001100000000000000000000000000000000000001110000000000000000000000000001100000000111000000000000000
17 - 000000000000001000000000000000000000000000111000000000000000000000011110000000000000010000000000111000000
18 - 000000000000000000000000011000000000000000000000000001111000000000000000000000001111000000000001100000000
19 - 001111000000000000000000000000000000000000001111000000000000000000000000111100000000000000001111000000000

20 - 111000000000000000000001000000000000000000111000000000000000000001000111100000000000000000000111100000000
21 - 000000000000000000000000000001100000000000110000000000000000000000001111100000000000000000000000111100000
22 - 000001100000000000000000000000000000000011110000000000000000000000011110000110000000000000000000111100000
23 - 000000000001100000000000000000000000000001100000000000111100000000000000000000000111110000000000011000000
24 - 000000000000000000000000110000000000000000110000000001100000000001100001100000000000110000000011110000000
25 - 111100000000000000000001000000000000000000111100000000000000000000100011110000000000000000000011111000000
26 - 000111100000000000000000000000000000000000001111000011000000000000000000000011110000001100001111000000000000
27 - 011000000000000000000000011000000000000000110000000000000111100000000000110000000001111000000000000111100000000
28 - 000001100000000000000000000000000000111100000000000000000011110000000000000011111000000000000000000011110
29 - 011111100000000000000000000000000000000000011111000000000000000000000001111100000000000000011111100000000

30 - 000000000000000000000001100000000000000111100000000000000000000000111111100000000000000000000001111100000
31 - 000000000011000000000000010000000000000011000000000011111100000000000000000000001111111000000000111000000
32 - 000000000010000000000000000011100000000000000000000011111100000000000000000000001111111000000000111000000
33 - 000000000000000010000000100000000000000011110000000000000000000000111111100000000000010000000011111000000
34 - 000011000000000000000000000000000011001110000000000000000001111000000000000011111100000000000000000011111
35 - 000100000000000000000001100000000000000011100100000000000000000000111111110000000000000000001111111000000
36 - 000000000000000000000010000001100000000011100000000000000000000000001111111110000000000000001001111110000000
37 - 001111000000000001000000000000000000000000011111100000000000000000000111111100000000010000011111110000000
38 - 111111000000000001000000000000000000000000011111100000000000000100000111111100000000000000111111110000000
39 - 011111000000000000001000000000000000000000011111100000000000000100000111111100000000000000011111110000000

40 - 000110000000000000000000000001110000000000000011111000000000000000000111111100000000000000111111110000000000
41 - 000000000000001100000000000000000000111100000000000000001111110000000000000001111111100000000000000011111
42 - 000000000000000001100000000001100000000011000000000001111111000000000000000000011111111100000000111100000
43 - 000000000000000000000001110000000000000000000111110000000000000000011111110000000000001111111111100000000
44 - 001110000000000000001100000000000000000111111000000000000000000110001111111110000000000000011111111100000000
45 - 000000000000000010000000111000000000000000001111110000000000000000111111100000000000001111111111100000000
46 - 011110000000000000110000000000000000000001111111000000000000001100001111111000000000000000111111111000000000
47 - 000010000000000000000011100000000000000111000111000000000000000011111111110000000000000111111111100000000
48 - 000000000000000000100001110000000000000001100011110000000000000011111111100000000000000111111111110000000
49 - 000000110000000000000000000111000001110000000000000011111111100000000000000111111111100000000000001111110

50 - 000000000000000111000000000000000000010000111111111000000000000000111111110000000000011111111111110000000
51 - 000000000000000001100000011000000000000000110011111100000000000000111111111100000000000011111111111100000000
52 - 000000110000000000000000000111100001100000000000000111111111100000000000000111111111111000000000001111110
53 - 000000000000000011100000000100000000000000110111111100000000000000111111111100000000000111111111111100000000
54 - 001100000000000011100000000000000000000011111111100000000000000111001111111110000000001111111111111000000
55 - 000000110000000000000000000111110001000000000000000111111111100000000000000111111111111100000000001111110
56 - 000000000000000111100000000000000000000001111111111000000000000001111111111000000000011111111111111000000000
57 - 000000000000000001110000010000000000000000111101111111000000000011111111111100000000000011111111111111000000000
58 - 000111110000000000000000000000000111000001111000000000000111111110000000000011111111111100000000000001111111110
59 - 000000000000000011110000000000000000000011111111110000000000000111111111111100000000000001111111111111000000

60 - 000000000000000000111000100000000000000011111011100000000000000111111111111110000000000011111111111111000000
61 - 000001100000000000000000000011111100000000000000000111111111111000000000000111111111111100000000001111111
62 - 000000010000000000111000000000000000000011111111100000000000000011111111111111000000000111011111111111000
63 - 000000000000000001110000000000000000000111111111100000000000000111111111111110000000000011111111111111100000
64 - 000011000000000000111000000000000000000011111111110000000000000111001111111111111000000111111111111111000
65 - 001111100000000000000000000000111110000011100000000000011111111100000000011111111111110000000000000011111111
66 - 001111110000000000000000000000111000000011110000000000111111110000000000111111111111111100000000000000111111110
67 - 000001100000000000111000000000000000000011111111110000000000000110011111111111111000000000111111111111111000
68 - 001111111000000000000000000000111000000011111100000111111110000000000000111111111111111100000000111111111100
69 - 000000000000000001110000000000000010001111111111000000000000011111111111111111000000001111111111111111110001

70 - 111111100000000000001100000000000000011111111110000000001111111000000111111111111111000000011111111111111100
71 - 011111100000000000000000000000111111000001110000000000111111111100000000111111111111111100000000001111111111
72 - 011111000000000000001110000000000000000111111111100000000011111100001111111111111110000000000111111111111100
73 - 001111000000000000111000000000000000000111111111110000000001111100001111111111111111000000000011111111111100
74 - 011111110000000000000000000011110000000001111000000011111111100000000011111111111111111000000000111111110
75 - 000000000000000001110000000001110000000000111111001110000000000011111111111110000000011111111111111111111000
76 - 011111110000000000000000000011100000000001111110000111111111000000000011111111111111111110000111111111100
77 - 011100000000000000111100000000000001111111110000000000000011111111100011111111111110000000111111111111111110
78 - 000000000000000001111000000011100000001111110001100000000000011111111111111110000000111111111111111111100
79 - 111100000000000000011110000000000000111111111100000000000111111111110001111111111111000001111111111111111110

80 - 111111100000000000000000000011111110000001110000000011111111100000000111111111111111110000000000111111111
81 - 000000000000000000111000000111100000000000111111000111100000000011111111111111100000011111111111111111111100
82 - 001100000000000000111100000000000000011111111100000001111111111110000000111111111111111000111111111111111100
83 - 001000000000000000000111100000000000011111111000000000011111111111111000001111111111111100001111111111111110
84 - 000000000000000001111000000000001111000011111100000001111111111111111000011111111111111111111000011111111
85 - 001100000000000000111100000000000000011111111111000000011111111111100000111111111111110011111111111111111100
86 - 000011111000000000000000000000111110000010000011111111111111000000001111111111111110001111111111111111111100
87 - 011111100000000000000000000000111111100000000011100001111111111111000000001111111111111111111111000111111111111
88 - 000000000000000000011100000011111110000011111000111110000000011111111111111111000011111111111111111111110
89 - 111000000000000000011111000000000000011111111111100000000111111111111000111111111111110001111111111111111110

90 - 011000000000000000011111000000000000011111111111100000000011111111111100001111111111111001111111111111111110
91 - 000000000000000000110000111111100000000000111001111111111100001111111111111111110000111111111111111111111110
92 - 000000110000000000000000000001111111110000000000111111111111111000011111111111111100011111111111111111111110
93 - 111110000000000000110000000000111111100000000111100001111111111111000001111111111111111111111111110011111111111
94 - 001111100000000000000000000001111111111000000000111001111111111111100000111111111111111111111111110011111111111
95 - 000000000001000001111000000000000000011111111111100001111011111111111000011111111111111111111111111111111110
96 - 000000000000000001111110000000000000011111111111100000011111111111111000011111111111111111111111111111111110
97 - 000000000000000000001100011111111100000000001110011111111110001111111111111111100011111111111111111111111
98 - 000000000001100001111000000000000000111111111111111000011000011111111111111111111111001111111111111111111111
99 - 000000010000000000000000000111111111010000000000011111111111111100011111111111111100111111111111111111111111

100 - 000011100000000000000000000000111111111100000000000111111111111111011100011111111111111011111111111111111111111
101 - 111000000000000000011110000001110000111100011111100001111111111111110001111111111111111111111111111111111111
102 - 000111100000000000000000000000111111111100000000001111111111111111111100001111111111111111111111111111111111111
103 - 000000000111110000011000000000000000111111111111110011111000001111111111111111111111111111111111111111111111
```

Figure 4 – *104 magic "high efficiency" poewr sinewaves.*

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Which gives us five equations to cancel out the third. Call your fifth harmonic bits *abccbaABCCBAabc*. Using 1-1/4 cycles per quarter word. Next, write out three equations that perfectly cancel your fifth. You have got eight equations total in fifteen unknowns. Next, use substitution to reduce this to one equation in seven unknowns. Which can drop us from 32768 states down to a mere 128!

Wait, there is more you can do. On a quarter cycle, these equations are *binaries*, whose only valid values are a zero or a one.

Ferinstance, if a binary equation of $U + V + W + X = Y + Z$ shows up, you do *not* need to check out all 64 cases. Go through the possibilities which allow *only* ones and zeros for any value, and you end up with only *fifteen* valid solutions.

You can safely ignore the others. Similarly, $J + K = L$ should only allow *three* valid solutions, not four.

By restricting your arithmetic to binary values, you can further reduce the 128 states. Even then, additional testing may still be needed.

At 60 bits, you'll find 31 different amplitudes having a zero third and a zero fifth. Of these, 010001110111111 is the "best" quarter cycle example. At 1.02 amplitude and a 3.4 percent seventh. The ninth is zero.

Even if you allow "weak" values of third and fifth, you still only get 48 or so total different amplitudes at 60 bits. But not all of those will be useful. Owing to uneven spacing or an occasional high seventh.

As you increase the total bits in your word your selection of *useful* solutions goes up at an infuriatingly slow rate. Thus the need to explore a lot of very long sequences.

In the case of 420 bits, we have a 105 bit quarter word. We can write 35 equations with no third, 21 with no fifth, and 15 with no seventh. 71 equations in 105 unknowns. Which reduces to something like 1 equation in 34 unknowns. Actually somewhat worse than this. Because of certain obscure cancellations. But there still are great heaping bunches left.

By using binary-only solutions, the total number of combinations can be further pared. The harmonic contribution of any chosen bit is a fixed numeric value. These values can be preplaced in a lookup table. Eliminating any on-the-fly trig or slow multiplications.

Two final reduction tools are to restrict the number of transitions or the number of ones in the words.

Selected for minimum total harmonic distortion with all odd harmonics present.
Only the first quadrant of each 420 bit word is shown…

```
00 - 000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
01 - 000000000000000000000000000000000000000000000000000000000001000000000000000000000000000000000000000000000
02 - 000000000000000000000000000000000000000000000000000000000000000000000000000000000000000100000000000000000
03 - 000000000000000000000000000000000000000000000000100000000000000000000000000000000000000100000000000000000
04 - 000000000000000000000000000000000000010000000000000000000000000001000000000000000000000100000000000000000
05 - 000000000000000000000000000000000000000001000000000000000000000000000000000010000000000000000000000000010000
06 - 000000000000000000000000000000000100000000000000000000000000010000000000000000000100000000000000001000000000
07 - 000000000000000000000000000001000000000000000000000100000000000000010000000000000100000000000001000000000
08 - 000000000000000000000000000000010000000000000000000000010000000000000000100000000000000100000000000000100
09 - 000000000000000000000000000100000000000000000000100000000000000100000000000010000000000100000000000100000
10 - 000000000000000000000000100000000000000000010000000000001000000000001000000000100000000100000000010000000
11 - 000000000000000000000000001000000000000000000100000000000001000000000001000000000100000000001000000000010
12 - 000000000000000000000001000000000000000001000000000000100000000010000000010000000001000000010000000010000
13 - 000000000000000000000100000000000000001000000000001000000001000000001000000010000000100000010000001000000
14 - 000000000000000000000010000000000000000100000000000010000000001000000001000000010000000100000001000000010
15 - 000000000000000000000100000000000000010000000000100000000100000001000000010000001000000100000010000001000
16 - 000000000000000000010000000000000010000000000100000001000000010000001000000100000100000100000100000100000
17 - 000000000000000000001000000000000001000000000010000000010000000100000001000000100000100000100000010000010
18 - 000000000000000000010000000000000100000000010000000010000001000000100000010000010000010000100000100000100
19 - 000000000000000000100000000000010000000001000000010000001000001000001000001000001000010000100001000010000
21 - 000000000000000000100000000000001000000000100000001000000010000010000010000010000010000100000100000100010
22 - 000000000000000001000000000000001000000000100000001000000100000100000100001000010000100001000010000100
23 - 000000000000000001000000000000100000000010000000100000100000100001000001000010001000010001000010001000
24 - 000000000000000001000000000000010000000001000000010000010000010000010000100001000010000100001000100001
25 - 000000000000000010000000000000100000000100000010000001000001000010000100001000100001000100001000100100
26 - 000000000000000010000000000010000000001000000100000100001000010000100001000100010001000100010001000
27 - 000000000000000010000000000001000000000100000010000010000100000100010000100001000100010000100010001
28 - 000000000000000100000000000010000000100000010000010000100001000010001000010001000100010010001000100100
29 - 000000000000000100000000000010000000100000100000100001000010001000010001000100100010001001000100100
30 - 000000000000000100000000000100000001000000100000100001000010000100010001000100010001000100100010010001
31 - 000000000000001000000000000100000010000001000010000100001000100010001000100010010001001000100100010
32 - 000000000000001000000000001000000100000100000100001000100010001000100010010001001000100100100010010
33 - 000000000000001000000000000100000001000001000010000100001000100010001000100010010001001001000100100100010001001
34 - 000000000000001000000000010000000100000100001000010001000100010001000100100010010010010001001001001001001010
35 - 000000000000010000000000010000001000001000010001000010001001000100100010010010010001010010010010010100
36 - 000000000000001000000000010000001000001000010000100010001000100010010010001001001001001001001001001
37 - 000000000000010000000000100000010000010000100001000100010010001001000100100100100100100100100100101001010
38 - 000000000000010000000000100000100000100001000100010001000100100010010010010010010010100100100101001010100
39 - 000000000000010000000000100000010000100001000010001000100010010001001001001001001001010010010010100101001
40 - 000000000000100000000010000001000001000100001000100010010010001001001001001010010010100100101001010010
41 - 000000000000100000000010000010000010001000010010001000100100100100100101001001010010100101001010010100
42 - 000000000000100000000010000001000010000100010001000100100010010010010010010100100101001010010100101001
43 - 000000000000100000000100000010000100001000100010001001001001001001001010010010100101001010010101001010
44 - 000000000000100000000100000100001000010001000100100010010010010100100101001010010101001010100101010100
45 - 000000000000100000000100000010000100010000100100010010010001010010010010100100101001010100101001010100101
46 - 000000000000100000000100000100001000010001001000100100100100101001001010010101001010100101010010101010
47 - 000000000001000000000100000100001000010001001000100100100101001010010100101010010101010010101010101010010
48 - 000000000000100000000100000010000100010001000100100100010100100100101001010010101001010100101010101001010101
49 - 000000000001000000000100000100001000100010001001001001001001010010100101010010101010100101010101010101010
50 - 000000000001000000000100000100010000100100010010010010010100101001010100101010101010100101010101010101010
51 - 000000000001000000000100000100001000100010010001001001001010010100101001010100101010101010100101010101010101
52 - 000000000001000000001000001000010001000100100010010010100100101001010100101010101010101010101010101010101010
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103 - 000000001000001000100100100101010101010101011011011011101110111101111111110111111111111111111111111111111
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Figure 5 – *104 magic "low distortion" power sinewaves.*

SOME RESULTS

A lot of juggling is still involved to come up with useful results. You want as many amplitudes as you can get that are evenly spaced. You want as few transitions as possible. You'll want low distortion. And you often have bunches of near misses, picking the best from a sorry lot.

At any rate, my selection for 104 efficient magic sinewaves that have low numbers of transitions appears in figure four. Harmonics 0,2,3,4,5,6, 7,8,9,10,12,14,15,16,18,20,21,22,24,25, 26,27,28 and 30 are *zero* for most of the sinewaves shown. The eleventh and thirteenth are often well under ten percent. *Before* filtering!

An occasional listing needing a seventh of less than one percent has been thrown in where it significantly reduced the transitions. And we had to borrow a few very low amplitudes from our upcoming Figure 5.

Amplitude spacing is monotonic and in roughly one percent steps.

There is significant step-to-step jitter owing to the random nature of the harmonic amplitude math.

The clocking rate for a 60 Hertz output is 25.2 kHz. But your actual transition frequencies are much less than this. For all but one value, the maximum number of transitions is 12 per quarter cycle or 48 total.

ANOTHER APPROACH

An interesting "transitions be damned" alternate is shown you in figure 5. Which gives you 104 magic sinewaves having very low distortion values. But with more transitions per cycle. And poorer efficiency.

But still significantly better and simpler than PWM.

All the odd harmonics here are present but very weak. Typically in the half percent range, at 46 or more decibels down. These values may be more useful when strong harmonic filtering is not a really great option. Such as an induction motor control running over a wide speed range.

A very sneaky method was used to find these values. Your trick is to make each bit contribute *as much as possible* to the desired fundamental. If the bit is so busy working on your fundamental, it might not have the energy or the inclination left over to generate strong harmonics!

You pick a desired fundamental amplitude. Then you simply start at the *middle* of your quarter cycle, picking out only those bits that take the biggest bite out of the remaining amplitude. But never exceeding it. This technique gives you amazingly good results. Very fast, too.

I've purposely shown all of these results in binary form. Viewing your actual sinewaves gives you insights into what is coming down. Do note particularly the efficient clumping of figure 4 and the progressive build of figure 5. Efficiency -*vs*- distortion.

If you select some more compact notation, be particularly careful that any hex leading "padding zeros" do not end up as portions of your actual output waveforms.

By the way, the general purpose PostScript language easily handles 420 bit words with aplomb. The key secret trick is to put your ones and zeros into a long string. You then can manipulate the string.

IMPROVING LOW AMPLITUDES

The results of either method are better for higher amplitudes. There's simply not enough ones to be placed in useful enough positions to give us really superb low amplitude results. Especially under fifteen percent or so. For some uses, lots of low values do not matter, because one quarter amplitude is only *seven* percent total power. One tenth amplitude is only *one* percent power.

If lots of low amplitudes end up essential, it seems best to combine the magic sinewaves with a second stepped-power selection scheme. Maybe using half versus full wave rectification to chop down the input voltage by two or four. Or switched taps. Or something similar.

Other possibilities are to forego quarter cycle symmetry or else try alternating amplitude states.

FOR MORE INFORMATION

To use these values, just stash them in a table and look them up as needed. Serial EEPROMS are a good

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choice in a PIC environment.

You'll find a *lot* of tradeoffs in figure 4, so consider what you see a "directors cut". By allowing 15 or 16 transitions per quarter cycle, you can get more steps with better spacing. At the price of lower efficiency. You can also permit some additional low harmonic distortion to pick up new candidate values. Or you select fewer amplitudes in order to improve both efficiency and distortion.

For multi-phase motors, some additional magic sequences can be suitably delayed. Everything shown is shiftable by thirds for three-phase power control applications.

Many thanks to math genius Jim Fitzsimons for his gracious help on this project. Especially for thinking out all of the hairy parts.

Once again, several files have now been posted up to the *Circuit Cellar* web site and also to my *www.tinaja.com* that give detailed magic sinewave results and explore other exciting options.

Included are thorough harmonic and transition analysis. Full raw data is also provided for making your own amplitude selections.

Yes, magic sinewave consulting and product development services are both definitely available. Start with *www.tinaja.com/magsn01.html*

Then go to the formal proposal at *www.tinaja.com/glib/msinprop.pdf*

Let's hear from you.

Microcomputer pioneer and guru Don Lancaster is the author of 35 books and countless tech articles. Don maintains his no-charge US tech helpline found at (520) 428-4073, besides offering all of his own books, reprints, and consulting services. Don also offers a free catalog full of his unique products and resource secrets. The best calling times are 8-5 on weekdays.

Don is the webmaster of his Guru's Lair site that you'll find at http://www.tinaja.com

You can also reach Don at his Synergetics, Box 809, Thatcher, AZ 85552. Or send any messages to his US Internet address you will find at don@tinaja.com

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