

QUADRATIC INTERPOLATION is a possible method of dramatically reducing the file size of stored table lookups. To work, the data has to be monotonic, well behaved, and only slowly changing.

Consider this series of **Magic Sinewave** data values...

[7948656 7940931 7933105 7925170 7917113 7908926
7900596 7892109 7883447 7874592 7865523]

The object of quadratic interpolation is to store only the first val(0) value in memory. Later calculating the others on an as needed basis.

Define a Δ increment and a δ increment change as follows...

$x(0) = \text{val}(0)$ from the table lookup

$$x(1) = x(0) + \Delta$$

$$x(2) = x(1) + \Delta + 1*\delta = \text{val}(0) + 2*\Delta + 1*\delta$$

$$x(3) = x(2) + \Delta + 2*\delta = \text{val}(0) + 3*\Delta + 3*\delta$$

$$x(4) = x(3) + \Delta + 3*\delta = \text{val}(0) + 4*\Delta + 6*\delta$$

$$x(5) = x(4) + \Delta + 4*\delta = \text{val}(0) + 5*\Delta + 10*\delta$$

$$x(6) = x(5) + \Delta + 5*\delta = \text{val}(0) + 6*\Delta + 15*\delta$$

$$x(7) = x(6) + \Delta + 6*\delta = \text{val}(0) + 7*\Delta + 21*\delta$$

$$x(8) = x(7) + \Delta + 7*\delta = \text{val}(0) + 8*\Delta + 28*\delta$$

$$x(9) = x(8) + \Delta + 8*\delta = \text{val}(0) + 9*\Delta + 36*\delta$$

$$x(10) = x(9) + \Delta + 9*\delta = \text{val}(0) + 10*\Delta + 45*\delta$$

Use Δ and δ to exactly fit midpoint val(5) and endpoint val(10) by solving...

$$\begin{aligned} \text{val}(5) - \text{val}(0) &= 5*\Delta + 10*\delta \\ \text{val}(10) - \text{val}(0) &= 10*\Delta + 45*\delta \end{aligned}$$

... simultaneously to get...

$$\begin{aligned} \Delta &= (\text{val}(10) - \text{val}(0) - 4.5*(\text{val}(5) - \text{val}(0)))/12.5 \\ \delta &= (\text{val}(10) - \text{val}(0) - 2*(\text{val}(5) - \text{val}(0)))/25 \end{aligned}$$

Or, for the above data $\Delta = -7651.97$ and $\delta = -146.964$. Using these values approximates the original sequence as...

[7948656 7941004 7933205 7925259 7917166 7908926
7900540 7892006 7883326 7874498 7865523]

... with errors that look like this...

[0 -73 -100 -89 -53 0 56 103 121 94 0]

In this case, the worst error is around twelve parts per million and around 4 PPM average. Note that the math changes wildly if you use fewer or more steps.

Matching other points instead of val(5) and val(10) may sometimes give an even better fit.

Additional support at <http://www.tinaja.com/magsn01.asp>