BY DON LANCASTER

# Understanding Active Filters

*Using op amps makes filter design easy and low-cost.*

FREQUENCY-SELECTIVE filters (high-pass, low-pass, or band-pass) can be either passive or active. The former is traditionally an inductor-capacitor circuit which, particularly at audio frequencies, can be cumbersome and expensive and have a response shape that is not as selective as desired. Active filters use conventional resistors and capacitors and operational amplifiers. They are low in cost, easy to tune, not sensitive to field and hum, small and light, and are not influenced by varying load and source impedances. In addition, active filters can be easily cascaded, so that a complex filter response can be broken down into simple factored blocks that do not interact.

Where are active filters used? Electronic music is one obviously important area. Here, active filters serve as modifiers of conventional instruments, to generate new sounds by way of formant synthesis and vcf (voltage controlled filter) techniques, and to generate the transient responses involved with bell and other percussion voices.

Biofeedback circuits that monitor brainwaves use ultra-low-frequency active filters to separate the alpha, beta, delta, and theta response waves. Active filters are also used in graphic equalizers to permit modifying the audio channel response to suit individual tastes or room acoustics. Microprocessor and computer-related uses of active filters include cassette tape sine-wave generators for data recording and transmission and reception of *modem* (modulator-demodulator) systems that send data over the phone lines.

Laboratory applications are widespread, ranging from ultra-low-frequency seismic and geophysical signal processing, to speech and hearing studies, and Doppler tracking of moving radar targets. Elaborate, general-purpose active filters are also available for many different lab situations where certain frequencies must be emphasized and others rejected or minimized. These same circuits can be converted into high-quality signal sources with external feedback.

The biggest users of active filters are probably engineers at the phone company. They developed most of the math concepts behind active filters and have an incredible variety of uses for them, ranging from multiplexing of phone conversations onto a common carrier to equalization of telephone lines.

Psychedelic lighting systems use active filters to pick up an audio signal, break it down into various frequency channels, and modulate colored lights or lasers on a multicolor dynamic display.

Actually, today you can use active filters for just about any frequency selective task you can dream up, ranging in frequency from a few hundredths of a hertz to several hundred kHz or more. The most common types of filter you'd be interested in are low-pass, band-pass, high-pass, universal, notch, and voltage-controlled filters. Now, let's take a detailed look at how you can build your own active filters.

**Low-Pass.** Active filters are normally broken down into building blocks that are simple and easy to tune. For fancier responses, you combine as many simple blocks as you need to get the overall desired result. One popular building block is called a *second-order section*. A second-order low-pass is pretty much flat in response up to a *cutoff frequency*. Above that, the response drops by one fourth each time you double the frequency. We say it has a cutoff slope of −12 dB per octave. A "mirror image" high-pass second-order section will have a complementary slope of +12 dB per octave, leveling off near the cutoff frequency and staying uniform for higher frequencies. Each of the second-order sections uses one or more operational amplifiers. For most lower frequency audio work, the 741 op amp is ideal. Improved 741's, particularly the duals and quads (4558 and 4136 are typical) are now available at low cost. Where you really need high-"Q" values or large signal swings at high frequencies, you can turn to a super 741 such as the LM318, with fifteen times the bandwidth and 150 times the slew rate of a stock 741. Or, if you're into very-low-frequency work, it pays to raise the impedance levels of your circuit as high as possible to get by with smaller valued capacitors. The FET or CMOS op amps are ideal for this, with the 3140 being a top choice for many low-cost applications.

A pair of second-order low-pass active filters, having a 1-kHz frequency are shown in Fig. 1. Each circuit is flat to near 1-kHz and then drops at −12 dB/octave well above 1-kHz. As the frequency increases, the response continues to die out.

The first circuit (Fig. 1A) is called the unity gain Sallen-Key circuit otherwise known as a VCVS or voltage controlled voltage-source filter. Since the op amp is used as a *source follower* (a noninverting amplifier with a gain of one, a high input impedance, and a low output impedance), an ordinary transistor emitter follower can be used just as well.

How does the circuit work? It looks at the overall mathematical *transfer func-*

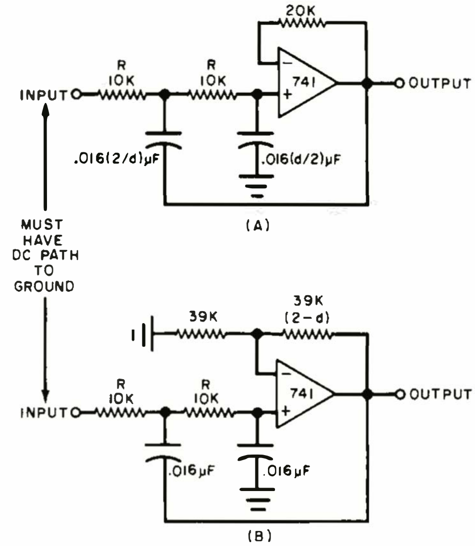


*Fig. 1. Unity gain (A) and equal component (B) Sallen-Key low pass filters. See table for values of d.*

**RESISTOR AND DAMPING VALUES FOR VARIOUS LOWPASS AND HIGHPASS RESPONSES**

| | First Section | | Second Section | | Third Section | |
|---|---|---|---|---|---|---|
| | Resistor R (kilohms) | Damping d | Resistor R (kilohms) | Damping d | Resistor R (kilohms) | Damping d |
| **Best Delay Low-pass** | | | | | | |
| 12 dB octave | 7.87 | 1.731 | — | — | — | — |
| 24 dB/octave | 6.98 | 1.916 | 6.19 | 1.241 | — | — |
| 36 dB/octave | 6.19 | 1.959 | 5.90 | 1.636 | 5.23 | 0.977 |
| **Flattest Low-pass** | | | | | | |
| 12 dB/octave | 10 | 1.414 | — | — | — | — |
| 24 dB/octave | 10 | 1.848 | 10.0 | 0.765 | — | — |
| 36 dB/octave | 10 | 1.932 | 10.0 | 1.414 | 10.0 | 0.518 |
| **1 dB Peak Low-pass** | | | | | | |
| 12 dB/octave | 11.5 | 1.045 | — | — | — | — |
| 24 dB/octave | 19.1 | 1.275 | 10.5 | 0.281 | — | — |
| 36 dB/octave | 28.8 | 1.314 | 13.7 | 0.455 | 10.2 | 0.125 |
| **Well-Damped High-pass** | | | | | | |
| 12 dB/octave | 12.7 | 1.731 | — | — | — | — |
| 24 dB/octave | 14.3 | 1.916 | 16.2 | 1.241 | — | — |
| 36 dB/octave | 16.2 | 1.959 | 16.9 | 1.636 | 19.1 | 0.977 |
| **Flattest High-pass** | | | | | | |
| 12 dB/octave | 10.0 | 1.414 | — | — | — | — |
| 24 dB/octave | 10.0 | 1.848 | 10.0 | 0.765 | — | — |
| 36 dB/octave | 10.0 | 1.932 | 10.0 | 1.414 | 10.0 | 0.518 |
| **1 dB Peak High-pass** | | | | | | |
| 12 dB/octave | 8.66 | 1.045 | — | — | — | — |
| 24 dB/octave | 5.23 | 1.275 | 9.53 | 0.281 | — | — |
| 36 dB/octave | 3.48 | 1.314 | 7.32 | 0.455 | 9.76 | 0.125 |

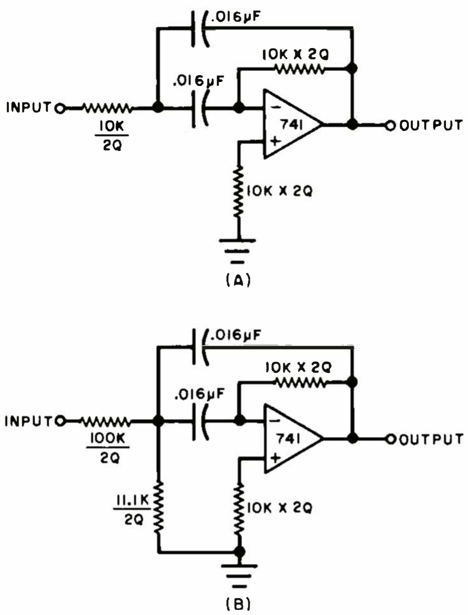


*Fig. 2. Multiple-feedback bandpass filter (A) is improved at (B) to have higher input impedance.*

*tion* for an inductor-capacitor-load circuit and synthesizes a similar result. So, while it does not actually replace the inductor, it's a simple matter with some fancy math to show that the circuit does everything that can be done with a passive inductor/capacitor filter and then some.

If the op amp weren't there, and if the first capacitor went to ground, we'd have an old-fashioned two-stage RC filter. This circuit has unity gain at very low frequencies (if not loaded), and a falloff at −12 dB/octave at very high frequencies. The problem is in-between where we'd like to have a sharp passband. Here the RC filter's response is very droopy and ill-defined.

Now, when the "ground" end of the first capacitor is connected to the *output* of the op amp, just enough energy is fed back from the power supply to simulate the energy storage in an inductor, and thus bolster the response as much as we want at the cutoff frequency. Very nicely, this feedback is localized *only* near the cutoff frequency. Why? Because the capacitor has too high a reactance to feed anything back at very low frequencies; and at very high frequencies, the output signal is too small to be worth feeding back. So, it's only near the cutoff frequency that the feedback has any appreciable effect.

Just how much energy do we want to feed back? This depends entirely on how much bolstering of the response we need near the cutoff frequency, and thus determines the cutoff response shape. The amount of feedback is called "*d*", short for *damping*. The *larger* the first capacitor is with respect to the second the *lower* the damping, and the *more peaked* the response. Values of *d* range from two down to zero. A damping of 2.00 is what we get with two cascaded but isolated RC sections. A *d* value of 1.73 will give the best possible transient and pulse response while a *d* of 1.41 gives the flattest possible amplitude and also a cutoff frequency that's exactly −3 dB down (0.707 voltage) from the fundamental. If we lower *d* further, we get a hump or peaking near the cutoff frequency. For instance, *d* values of 1.045, 0.895, and 0.767 correspond to humps of one, two, and three decibels respectively. If *d* ever hits zero, we get infinite peaking, otherwise known as an output with no input, or an oscillator.

To build the Fig. 1A circuit, we must decide what the damping is going to be, and then calculate the two capacitor values. For a flattest amplitude filter (also called a Butterworth), *d* will equal 1.41, and the left capacitor will be 0.02 µF and the right capacitor will be 0.01 µF, rounded off to stock values.

How do we change frequency? By changing either the capacitors or the resistors marked "R" or both. The only thing NOT allowed is to change the ratio of the two resistors (from 1:1) or the ratio of the two capacitors (from $4/d^2$) The *product* of the resistors and capacitors sets the frequency. The *ratio* of the capacitors sets the damping figure.

If the capacitor values are doubled, the cutoff frequency drops to 500 Hz. If the resistance values are doubled, the cutoff frequency also drops to 500 Hz. Do both and the frequency drops to 250 Hz and so on. The capacitors can be switched in steps and a dual potentiometer used to change resistance for a 10:1 frequency change.

By the way, note that the frequency varies inversely with the potentiometer settings. This will give you a dial that's very cramped at one end and nonlinear. Two ways to beat this problem are to use pots with reverse log tapers or to use pots with standard audio log tapers but put the dial on the pot shaft and the pointer on the panel, instead of vice versa. Selector switches and stepped resistor values provide another route to frequency selection and usually offer more precise control than ganged pots. Frequency steps can be in a linear or log arrangement.

Polystyrene capacitors are excellent for active filter use, but you have to keep them away from solvents and be careful not to nick them with a soldering iron. More expensive mica and Mylar capacitors can also be used. Under no circumstances should a disc or an electrolytic capacitor be used for filters.

There are one or two details that can cause trouble if you don't watch for them. With this circuit or any other lowpass filter, you have to bias the op amp's inputs in some way. This is usually done through the source, so there has to be a low-resistance dc return path through the source to ground. The source impedance, dc and otherwise, should be well below 10,000-ohms if it's not going to change the response. A second detail is to note that this is a true lowpass filter, so it also passes dc. Any bias, dc level, or offset voltage at the input goes on to the output, and if too large, can saturate the amplifier or limit the dynamic range. This effect can be eliminated by putting a blocking capacitor on the input, but you still have to bias your op amp. The 20,000-ohm resistor connected to the negative input isn't critical, and usually it is picked for minimum op amp offset.

While the Fig 1A circuit is simple and easy to build, we can do better. The capacitor values are hard to calculate and tend to spread widely for low *d* values, thus damping is hard to adjust. There's also no easy way to switch from high-

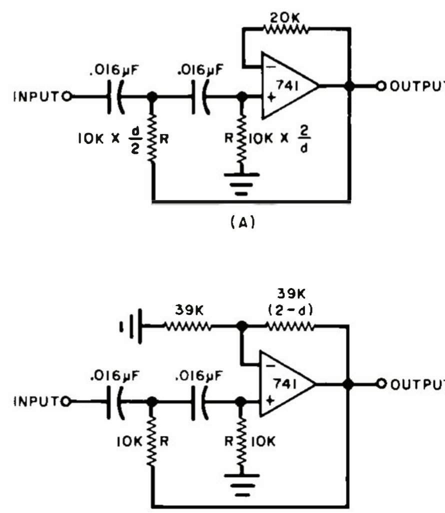


*Fig. 3. High-pass filters: unity gain (A) and equal component (B).*

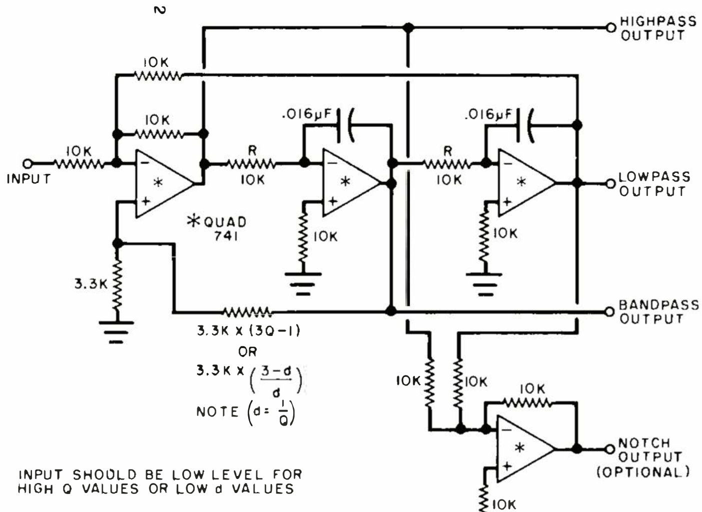


*Fig. 4. Universal, state-variable filter has low-pass, band-pass, high-pass, and notch outputs.*

pass to low-pass just by rearranging components.

If you go through the Sallen-Key math in detail (a very ugly process), it turns out that there is one magic value of op amp gain that solves all these problems. This is the ultra-simple and practically unknown *equal component value Sallen-Key filter* shown in Fig. 1B. The magic gain value is $3-d$, which means that you trim the damping by trimming the gain. The *R* resistors and the capacitors are identical values and are changed in pairs to change frequency. As an added feature, it can be changed to a high-pass characteristic with identical response simply by interchanging the resistors and capacitors.

One new detail to watch is that the feedback resistor must be held lower than the 78,000-ohm value that corresponds to a $d = 0$ oscillator. Fortunately, the *d* values shown here are normally well away from this danger zone, and the gain is easily set by the ratio of two resistors.

**FOR MORE INFORMATION**

Here are some good sources of information on active filters:

*The Active Filter Cookbook*, #21168, Howard W. Sams, Indianapolis, IN 46206, (1975).

"A Practical Method of Designing RC Active Filters," *IRE Transactions*, CT-2, March 1955.

"State Variable Synthesis for Insensitive Integrated Circuit Transfer Functions," *IEEE Journal*, SC-2, September 1967.

The first of these has the most detail on circuits, background math, and tuning techniques (for these and other circuits), along with many response curves, rip-off circuits, and detailed band-pass design information. The second and third references are theoretical "horses mouth" source documents covering the theory behind Sallen-Key and State-Variable filters.

**Band-pass.** Sallen-Key techniques don't really make good band-pass filters, so we go to the *multiple feedback* filters shown in Fig. 2. Usually, we are involved with such low *d* values in a bandpass filter that we use its inverse or Q instead. The Q is simply the ratio of the bandwidth to the center frequency. The circuit of Fig. 2A has a gain of $-2Q^2$ at resonance (the minus means a 180-degree phase shift), and a resonance frequency of 1 kHz.

The circuit is tuned by changing the values of the resistors or the capacitors, but, once again, both resistors and both capacitors are kept at fixed ratios.

The op amp gain should be at least $20Q^2$ at the operating frequency, so this particular circuit works best with lower Q values and lower resonance frequencies. You also tend to get a wide resistor spread with high Q values so this circuit is best used for Q values of 20 or less. At resonance, the gain is very high, so be sure to limit the size of the input signals so the op amp doesn't clip or saturate.

The extra resistor added to Fig. 2B raises the input impedance and drops the gain. However, it still has a respectable gain of $-Q^2/5$ and ten times the input impedance of the earlier filter circuit.

**High-pass.** The Sallen-Key circuits can be used for high-pass by making them mirror images of the low-pass. These are shown in Fig. 3. Note that the unity gain version (3A) now has resistor ratios set by the damping and 1:1 capacitor ratios, so there is no way to switch the same parts around for identical low-pass and high-pass responses. The equal component value circuit of Fig. 3B doesn't have this problem and we get from high-pass to low-pass with 4pdt switching. Since there is an internal dc bias path, we no longer have to worry about providing a dc return through the source.

High-pass filters are inherently noisier than low-pass ones because they emphasize transients, and pass harmonics of supposedly rejected wave forms. Certain circuits tend to reduce the stability margins of the internal op amp compensation. So, rarely will you get a really "clean" high-pass output from a filter, active or passive. Note also that the op amp sets an upper frequency limit and you have to save enough "daylight" between the desired cutoff frequency and the op amp's cutoff frequency to have a passband left.

Sometimes the capacitor values of a low-frequency active filter (high-pass or

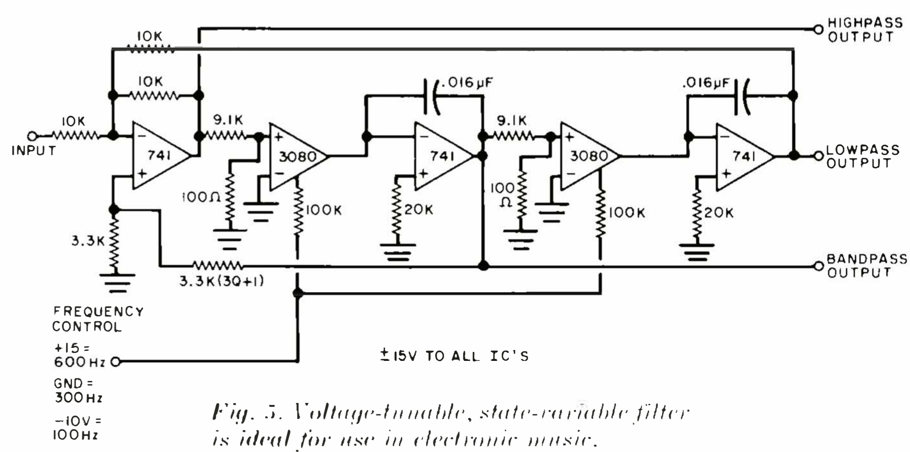


*Fig. 5. Voltage-tunable, state-variable filter is ideal for use in electronic music.*

otherwise) get too large and too expensive. This can be avoided by raising the impedance of the circuit suitably. For instance, to raise the impedance by a factor of ten *multiply* all resistors by ten and *divide* all capacitors by ten. These higher impedance circuits tend to be more offset-sensitive and should be used only when capacitor size is a serious problem.

**Universal Filters.** These are also called *state variable filters*, and they take three or four op amps per second-order section, often in a quad package, and use more resistors than the simpler circuits. However, they are vastly better. Universal filters have three, and sometimes more, simultaneous outputs—low-pass, band-pass, high-pass, and an optional notch output. They are easily used with Q values of 500 or more and don't tax the frequency limits of the op amps very heavily at all. They easily realize ultra-low *d* values without stability problems, they are easy to voltage tune, and they are very easy to switch from high-pass to band-pass to low-pass. About their only limitation is that a lot of parts are required in systems with fancy filter responses and multiple channels.

One universal filter is shown in Fig. 4. It is tuned by changing the *R* resistors or the capacitors. Once again, the resistors should be identical and the capacitors identical at all times. The Q or *d* is set with the feedback resistor as shown, while op amp gain at the cutoff frequency should be 3Q or better. Note that Q or *d* is easy to adjust independently. We can also design to different values of circuit gain, but this involves some non-obvious resistor calculations on the first stage. For completely independent gain, damping, and frequency another op amp can be added.

The low-pass, band-pass, and high-pass outputs are progressively phase-shifted by 90 degrees at the cutoff frequency. We can build quadrature art systems by routing the LP and BP outputs to a scope or plotter and inputting interesting audio signals to the filter. Since the circuit gain at resonance is Q, be sure to limit input signals to a suitably small size.

This circuit is really an analog computer that models a rusty pendulum. With an infinite Q resistor ($d = 0$), there is no damping (an oscillator). The Q resistor adds rust, or damping, to the pendulum.

The notch output shown has nothing to do with the rest of the circuit and can be left off if not desired. This output produces a zero output at resonance and a notch width proportional to the circuit Q. The notch works by summing the low-pass and high-pass outputs which cancel at the resonance frequency. If one of these input resistors is changed, the notch can be moved either side of the resonance frequency. This is a powerful class of filters called *Cauer or Elliptical* filters that strongly reject signals immediately outside the passband.

**Cascading.** Two second-order sections can be connected together to build a fourth, and three to get a sixth, but *the damping and frequency values must be watched if a useful overall response is to be obtained*. For instance, we've seen how a maximally flat second-order section is built with a *d* value of 1.41. But cascade three of these and what was a −3 decibel cutoff frequency is now a very droopy −9 decibels and no longer flat at all.

The Table shows the correct damping and frequency values for high-pass and low-pass filters of second, fourth, and sixth order. The shapes selected are for the best delay, the flatest amplitude, and a slightly peaked response. These are called the *Bessell*, *Butterworth*, and *One Decibel Chebycheff* responses. The cutoff frequency of all values, defined to three decibels below peak response is 1 kHz. The circuits can be tuned to any other frequency by the previous techniques we've looked at, but all cascaded sections must be changed by the same amount. While five-percent resistor and capacitor values are usually more than adequate for these circuits, values correct to one percent are indicated in the Table.

**Voltage Control.** To voltage control a universal filter, replace the fixed or variable frequency determining *R* resistors with something that looks like an electrically variable resistor. One very good choice is the CA3080 transconductance amplifier, and a voltage controlled universal filter can be built as shown in Fig. 5. This circuit provides a linear voltage versus frequency control; and frequency ranges of 100:1 and even 1000:1 are possible with careful design. One important design detail is to keep the input voltage on the 3080 positive input to 100 millivolts or less peak-to-peak for good linearity. ◇