Extended Resonance Curves

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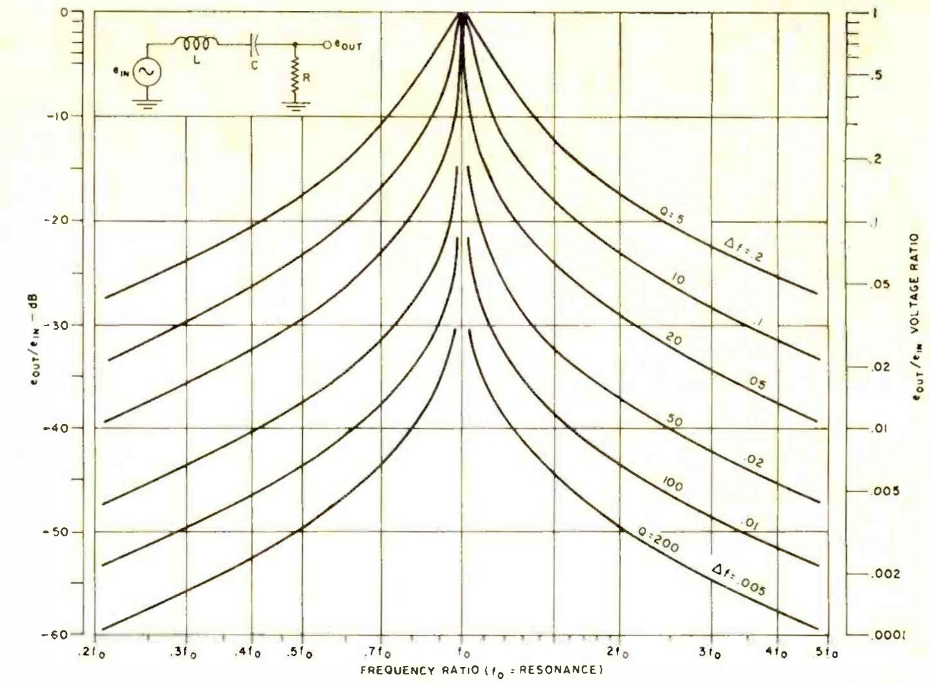

Fig. 1. These curves can be used to predict resonant circuit performance from one-fifth to five times resonant frequency.

Extension of universal resonance curves permits prediction of performance at frequencies well off the actual resonance.

THE universal resonance curves given in most circuit theory books are rather difficult to read when dealing with high-"Q" circuits or with frequencies well off resonance. Presented here is a family of extended resonance curves that accurately predict the performance of any series *RLC* resonant circuit or any high-"Q" parallel-resonant circuit for frequencies ranging from one-fifth to five times the resonant frequency.

The basic curves, which are constructed for various values of "Q", are shown above. Although no actual frequency is indicated for horizontal scale, f_o (scale center) is the center frequency selected.

The curves may be used to predict the harmonic rejection of a simple "one-pole" filter, as well as to indicate the selectivity that a tuned circuit can provide. The curves will also show whether a simple tuned trap will solve a given rejection or filter problem, or whether a more complex filter structure is called for.

The rejection of a series *RLC* circuit at any frequency is given by the formula: *insertion loss* $= 20\log_{10}\{1 + jQ[(\omega^2 - 1)/\omega]\}$ if the load resistance and angular resonant frequency are normalized to unity. The same relation also holds true for those parallel-resonant circuits having a "Q" that exceeds 10.

The extended resonance curves are simply a plot of this equation for various values of frequency, insertion loss, and "Q", and they appear in Fig. 1. The frequency is shown on the horizontal scale and is given in terms of the *ratio* of the frequency in question to the resonant *RLC* frequency. Insertion loss is plotted vertically and is given both in decibels (on the left) and in output/input voltage ratios (on the right). Various values of "Q" appear as curve parameters. The "Q" specified is the *loaded* circuit "Q" that includes *both* the load resistance and any circuit losses associated either with the particular inductor or capacitor being used. The bandwidth of each "Q" curve is also given, expressed in terms of the frequency difference between the —3 dB points as a fraction of the center frequency. For high-"Q" circuits, the bandwidth is very nearly centered about the resonant frequency of the tuned circuit.

Using the Curves

Some examples of actual use will demonstrate the value of these curves.

Example 1. What is the minimum "Q" required to guarantee a 40-dB rejection of the second harmonic of a 100-kHz signal? What will the filter bandwidth be?

Entering the frequency axis at $2f_o$ and the loss axis (left one) at —40 dB, note that a "Q" of approximately 60 is required. The bandwidth will equal 1/"Q", or 0.0167, which, when multiplied by the 100-kHz resonant frequency, will be 1.67 kHz, half on each side of 100 kHz. The —3 dB points will then be very close to 99.2 and 100.8 kHz. Any signals not between these two frequencies will be rejected by at least 3 dB. To realize this particular filter, 1% components will be required, unless a tuning provision is made.

Example 2. What rejection will a 2-MHz, "Q" = 20 parallel-tuned circuit give to 1.0, 2.0, 3.0, and 16.0 MHz?

1.0 MHz is 1.0/2.0 or 0.5 times the resonant frequency. Entering the curves at $0.5f_o$ and "Q" = 20 produces 29 dB rejection. Since 2.0 MHz is the resonant frequency, the rejection will be zero at this frequency. 3.0 MHz is a frequency ratio of 3.0/2.0 or $1.5f_o$, for which we read an insertion loss of 24 dB, or a voltage ratio of approximately 0.063.

16 MHz gives a normalized frequency of $8f_o$ which is beyond the range of the curves, but we can note that all the curves are smoothly falling at 6 dB per octave for frequencies well above, or well below, resonance. The loss at $8f_o$ will be 6 dB more than the loss at $4f_o$, or 37 + 6 = 43 dB of insertion loss.

Actual values of inductance and capacitance required in a series filter are obtained by specifying the load resistance and then using the formulas $L = RQ/6.28f$ and $C = 0.0253/f^2L$. The unloaded "Q" of the inductor that is employed must be significantly higher than the loaded circuit "Q". ▲