PROBLEM:

Create a minimal pulse steplock sequence that generates a fundamental sinewave of 0.53 amplitude but has zero harmonics through the sixteenth.

SOLUTION:

By working in quadrants, all even harmonics will automatically be forced to zero. Eight pulse edges will be needed to control the fundamental and the first seven odd harmonics (3, 5, 7, 9, 11, 13, 15). For eight equations in eight unknowns.

Write the eight harmonic equations using Chebyshev polynominals…

$$
\begin{array}{l}
\mathbf{T_1}(P1S) - \mathbf{T_1}(P1E) + ... + \mathbf{T_1}(P4S) - \mathbf{T_1}(P4E) = 0.53 * \pi/4 \\
\mathbf{T_3}(P1S) - \mathbf{T_3}(P1E) + ... + \mathbf{T_3}(P4S) - \mathbf{T_3}(P4E) = 0 \\
\mathbf{T_5}(P1S) - \mathbf{T_5}(P1E) + ... + \mathbf{T_5}(P4S) - \mathbf{T_5}(P4E) = 0 \\
\quad .... \qquad\qquad .... \qquad\qquad .... \\
\mathbf{T_{15}}(P1S) - \mathbf{T_{15}}(P1E) + ... + \mathbf{T_{15}}(P4S) - \mathbf{T_{15}}(P4E) = 0
\end{array}
$$

Where P1S is the cosine of the first pulse start angle, P1E is the cosine of the first pulse end angle, $\mathbf{T_1}$ is a first order Chebyshev polynomial (used to define the fundamental), $\mathbf{T_3}$ is a third order Chebyshev Polynomial (used to define the third harmonic), etc… That π/4 is a Fourier Series scaling factor.

By adding or subtracting previous line multiples, the equations can be reduced to this elegantly simple form…

$$
\begin{array}{l}
(P1S)^1 - (P1E)^1 + ... + (P4S)^1 - (P4E)^1 = 0.53 * \pi/4 \\
(P1S)^3 - (P1E)^3 + ... + (P4S)^3 - (P4E)^3 = 0.53 * 3\pi/16 \\
(P1S)^5 - (P1E)^5 + ... + (P4S)^5 - (P4E)^5 = 0.53 * 5\pi/32 \\
\quad .... \qquad\qquad .... \qquad\qquad .... \qquad\qquad ....
\end{array}
$$

Successive vertical amplitude values are scaled by the bizarre power sequence (3/4)(5/6)(7/8)(9/10)… Solution of these equations using my fast and simple PostScript iterative approximation procs gives you these values…

P1 start: 17.9125	P1 end: 21.4007
P2 start: 36.1121	P2 end: 42.7902
P3 start: 54.8818	P3 end: 64.1028
P4 start: 74.4503	P4 end: 85.1345