The problem is to generate an approximation to a sine wave with eight Bézier curves.

There are nine points the curves must pass thru. $[x_0, y_0]$, $[x_3, y_3]$, $[x_6, y_6]$, $[x_9, y_9]$, $[x_{12}, y_{12}]$, $[x_{15}, y_{15}]$, $[x_{18}, y_{18}]$, $[x_{21}, y_{21}]$, $[x_{24}, y_{24}]$,

There are 16 control points to find. Solve for the first curve.

$$q = a_0\left(3\ t^2 - 1\right) + b_0\left(2\ t - 1\right) - x_0 + x_3$$

$$p = 6\ a_0\ t + 2\ b_0$$

$$p_0 = 2\ b_0$$

$$q = a_0\left(3\ t^2 - 1\right) + p_0\left(t - \frac{1}{2}\right) - x_0 + x_3$$

$$p = 6\ a_0\ t + p_0$$

$$p_3 = 6\ a_0 + p_0$$

$$a_0 = \frac{p_3 - p_0}{6}$$

$$6\ q = 3\ t^2\left(p_3 - p_0\right) + 6\ p_0\ t - 2\ p_0 - p_3 + 6\left(x_3 - x_0\right)$$

$$p = p_0\left(1 - t\right) + p_3\ t$$

$$6\ q_0 = -2\ p_0 - p_3 + 6\left(x_3 - x_0\right)$$

$$6\ q_3 = p_0 + 2\ p_3 + 6\left(x_3 - x_0\right)$$

The second curve is next.

$$6\ q_3 = -2\ p_3 - p_6 + 6\left(x_6 - x_3\right)$$

$$p_3 = -\frac{p_0}{4} - \frac{p_6}{4} + \frac{3\,x_0}{2} - 3\ x_3 + \frac{3\,x_6}{2}$$

The third is next.

$$p_6 = -\frac{p_3}{4} - \frac{p_9}{4} + \frac{3\,x_3}{2} - 3\ x_6 + \frac{3\,x_9}{2}$$

$$p_9 = -\frac{p_6}{4} - \frac{p_{12}}{4} + \frac{3\,x_6}{2} - 3\ x_9 + \frac{3\,x_{12}}{2}$$

$$p_{12} = -\frac{p_9}{4} - \frac{p_{15}}{4} + \frac{3\,x_9}{2} - 3\ x_{12} + \frac{3\,x_{15}}{2}$$

$$p_{15} = -\frac{p_{12}}{4} - \frac{p_{18}}{4} + \frac{3\,x_{12}}{2} - 3\ x_{15} + \frac{3\,x_{18}}{2}$$

$$p_{18} = -\frac{p_{15}}{4} - \frac{p_{21}}{4} + \frac{3\,x_{15}}{2} - 3\ x_{18} + \frac{3\,x_{21}}{2}$$

$$p_{21} = -\frac{p_{18}}{4} - \frac{p_{24}}{4} + \frac{3\,x_{18}}{2} - 3\ x_{21} + \frac{3\,x_{24}}{2}$$

There are seven equations to solve. For a sine curve $p_0 = 0$, $p_{12} = 0$, and $p_{24} = 0$. That and other information about the sine curve reduces the problem to three equations for the first half and three equations for the second half.

$$p_3 = \frac{45\,x_0}{28} + \frac{3\,x_{12}}{28} - \frac{51\,x_3}{14} + \frac{18\,x_6}{7} - \frac{9\,x_9}{14}$$

$$p_6 = -\frac{3\,x_0}{7} - \frac{3\,x_{12}}{7} + \frac{18\,x_3}{7} - \frac{30\,x_6}{7} + \frac{18\,x_9}{7}$$

$$p_9 = \frac{3\,x_0}{28} + \frac{45\,x_{12}}{28} - \frac{9\,x_3}{14} + \frac{18\,x_6}{7} - \frac{51\,x_9}{14}$$

There are three equations for the second half.

$$p_{15} = \frac{45\,x_{12}}{28} - \frac{51\,x_{15}}{14} + \frac{18\,x_{18}}{7} - \frac{9\,x_{21}}{14} + \frac{3\,x_{24}}{28}$$

$$p_{18} = -\frac{3\,x_{12}}{7} + \frac{18\,x_{15}}{7} - \frac{30\,x_{18}}{7} + \frac{18\,x_{21}}{7} - \frac{3\,x_{24}}{7}$$

$$p_{21} = \frac{3\,x_{12}}{28} - \frac{9\,x_{15}}{14} + \frac{18\,x_{18}}{7} - \frac{51\,x_{21}}{14} + \frac{45\,x_{24}}{28}$$

The values of x are equally spaced.

$$x_3 = x_0 + \frac{\pi}{4}$$

$$x_6 = x_0 + \frac{\pi}{2}$$

$$x_9 = x_0 + \frac{3\,\pi}{4}$$

$$x_{12} = x_0 + \pi$$

$$[p_3 = 0,\, p_6 = 0,\, p_9 = 0]$$

$$[p_{15} = 0,\, p_{18} = 0,\, p_{21} = 0]$$

$$q_0 = \frac{\pi}{4}$$

$$q_3 = \frac{\pi}{4}$$

$$3\,(x_1 - x_0) = \frac{\pi}{4}$$

$$x_1 = \frac{\pi}{12} + x_0$$

$$3\,(x_3 - x_2) = \frac{\pi}{4}$$

$$x_2 = x_3 - \frac{\pi}{12}$$

The other control points have similar values. Now we repeat the process for the y values.

$$p_3 = \frac{45\,y_0}{28} + \frac{3\,y_{12}}{28} - \frac{51\,y_3}{14} + \frac{18\,y_6}{7} - \frac{9\,y_9}{14}$$

$$p_6 = -\frac{3\,y_0}{7} - \frac{3\,y_{12}}{7} + \frac{18\,y_3}{7} - \frac{30\,y_6}{7} + \frac{18\,y_9}{7}$$

$$p_9 = \frac{3\,y_0}{28} + \frac{45\,y_{12}}{28} - \frac{9\,y_3}{14} + \frac{18\,y_6}{7} - \frac{51\,y_9}{14}$$

$$y - y_0 = \sin\left(x - x_0\right)$$

$$y_3 - y_0 = \sin\left(x_3 - x_0\right)$$

$$y_3 - y_0 = \sin\left(\frac{\pi}{4}\right)$$

$$y_3 - y_0 = \frac{\sqrt{2}}{2}$$

$$y_6 - y_0 = \sin\left(\frac{\pi}{2}\right)$$

$$y_6 - y_0 = 1$$

$$y_9 - y_0 = \sin\left(\frac{3\,\pi}{4}\right)$$

$$y_9 - y_0 = \frac{\sqrt{2}}{2}$$

$$y_{12} - y_0 = 0$$

$$y_{15} - y_0 = -\frac{\sqrt{2}}{2}$$

$$y_{18} - y_0 = -1$$

$$y_{21} - y_0 = -\frac{\sqrt{2}}{2}$$

$$y_{24} - y_0 = 0$$

$$\left[p_3 = \frac{18}{7} - \frac{15\,\sqrt{2}}{7},\ p_6 = \frac{18\,\sqrt{2}}{7} - \frac{30}{7},\ p_9 = \frac{18}{7} - \frac{15\,\sqrt{2}}{7}\right]$$

$$p_{15} = \frac{45\,y_{12}}{28} + \frac{3\,y_{24}}{28} - \frac{51\,y_{15}}{14} + \frac{18\,y_{18}}{7} - \frac{9\,y_{21}}{14}$$

$$p_{18} = -\frac{3\,y_{12}}{7} - \frac{3\,y_{24}}{7} + \frac{18\,y_{15}}{7} - \frac{30\,y_{18}}{7} + \frac{18\,y_{21}}{7}$$

$$p_{21} = \frac{3\,y_{12}}{28} + \frac{45\,y_{24}}{28} - \frac{9\,y_{15}}{14} + \frac{18\,y_{18}}{7} - \frac{51\,y_{21}}{14}$$

$$\left[p_{15} = \frac{15\,\sqrt{2}}{7} - \frac{18}{7},\ p_{18} = \frac{30}{7} - \frac{18\,\sqrt{2}}{7},\ p_{21} = \frac{15\,\sqrt{2}}{7} - \frac{18}{7}\right]$$

$$6\ q_0 = -2\ p_0 - p_3 + 6\,(y_3 - y_0)$$

$$6\ q_3 = p_0 + 2\ p_3 + 6\,(y_3 - y_0)$$

$$6\,(3\,(y_1 - y_0)) = -2\ p_0 - p_3 + 6\,(y_3 - y_0)$$

$$y_1 = -\frac{p_0}{9} - \frac{p_3}{18} + \frac{2\,y_0}{3} + \frac{y_3}{3}$$

$$6\,(3\,(y_3 - y_2)) = p_0 + 2\ p_3 + 6\,(y_3 - y_0)$$

$$y_2 = -\frac{p_0}{18} - \frac{p_3}{9} + \frac{y_0}{3} + \frac{2\,y_3}{3}$$

$$\left[y_1 = -\frac{p_0}{9} - \frac{p_3}{18} + \frac{2\,y_0}{3} + \frac{y_3}{3},\ y_2 = -\frac{p_0}{18} - \frac{p_3}{9} + \frac{y_0}{3} + \frac{2\,y_3}{3}\right]$$

$$\left[y_4 = -\frac{p_3}{9} - \frac{p_6}{18} + \frac{2\,y_3}{3} + \frac{y_6}{3},\ y_5 = -\frac{p_3}{18} - \frac{p_6}{9} + \frac{y_3}{3} + \frac{2\,y_6}{3}\right]$$

$$\left[y_7 = -\frac{p_6}{9} - \frac{p_9}{18} + \frac{2\,y_6}{3} + \frac{y_9}{3},\ y_8 = -\frac{p_6}{18} - \frac{p_9}{9} + \frac{y_6}{3} + \frac{2\,y_9}{3}\right]$$

$$\left[y_{10} = -\frac{p_9}{9} - \frac{p_{12}}{18} + \frac{2\,y_9}{3} + \frac{y_{12}}{3},\ y_{11} = -\frac{p_9}{18} - \frac{p_{12}}{9} + \frac{y_9}{3} + \frac{2\,y_{12}}{3}\right]$$

$$\left[y_{13} = -\frac{p_{12}}{9} - \frac{p_{15}}{18} + \frac{2\,y_{12}}{3} + \frac{y_{15}}{3},\ y_{14} = -\frac{p_{12}}{18} - \frac{p_{15}}{9} + \frac{y_{12}}{3} + \frac{2\,y_{15}}{3}\right]$$

$$\left[y_{16} = -\frac{p_{15}}{9} - \frac{p_{18}}{18} + \frac{2\,y_{15}}{3} + \frac{y_{18}}{3},\ y_{17} = -\frac{p_{15}}{18} - \frac{p_{18}}{9} + \frac{y_{15}}{3} + \frac{2\,y_{18}}{3}\right]$$

$$\left[y_{19} = -\frac{p_{18}}{9} - \frac{p_{21}}{18} + \frac{2\,y_{18}}{3} + \frac{y_{21}}{3},\ y_{20} = -\frac{p_{18}}{18} - \frac{p_{21}}{9} + \frac{y_{18}}{3} + \frac{2\,y_{21}}{3}\right]$$

$$\left[y_{22} = -\frac{p_{21}}{9} - \frac{p_{24}}{18} + \frac{2\,y_{21}}{3} + \frac{y_{24}}{3},\ y_{23} = -\frac{p_{21}}{18} - \frac{p_{24}}{9} + \frac{y_{21}}{3} + \frac{2\,y_{24}}{3}\right]$$

Calculate the y values of the control points:

$$y_1 = y_0 + \frac{2\sqrt{2}}{7} - \frac{1}{7}$$

$$y_2 = y_0 + \frac{4\sqrt{2}}{7} - \frac{2}{7}$$

$$y_4 = y_0 + \frac{3\sqrt{2}}{7} + \frac{2}{7}$$

$$y_5 = y_0 + 1$$

$$y_7 = y_0 + 1$$

$$y_8 = y_0 + \frac{3\sqrt{2}}{7} + \frac{2}{7}$$

$$y_{10} = y_0 + \frac{4\sqrt{2}}{7} - \frac{2}{7}$$

$$y_{11} = y_0 + \frac{2\sqrt{2}}{7} - \frac{1}{7}$$

$$y_{13} = y_0 - \frac{2\sqrt{2}}{7} + \frac{1}{7}$$

$$y_{14} = y_0 - \frac{4\sqrt{2}}{7} + \frac{2}{7}$$

$$y_{16} = y_0 - \frac{3\sqrt{2}}{7} - \frac{2}{7}$$

$$y_{17} = y_0 - 1$$

$$y_{19} = y_0 - 1$$

$$y_{20} = y_0 - \frac{3\sqrt{2}}{7} - \frac{2}{7}$$

$$y_{22} = y_0 - \frac{4\sqrt{2}}{7} + \frac{2}{7}$$

$$y_{23} = y_0 - \frac{2\sqrt{2}}{7} + \frac{1}{7}$$

Here are the eight curves with control points.